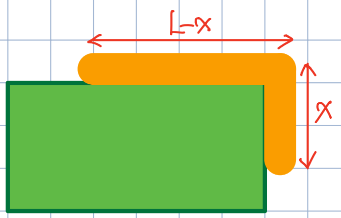
String Problems

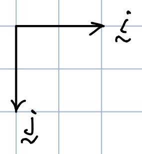
density of string : ρ (per unit length)
length : L



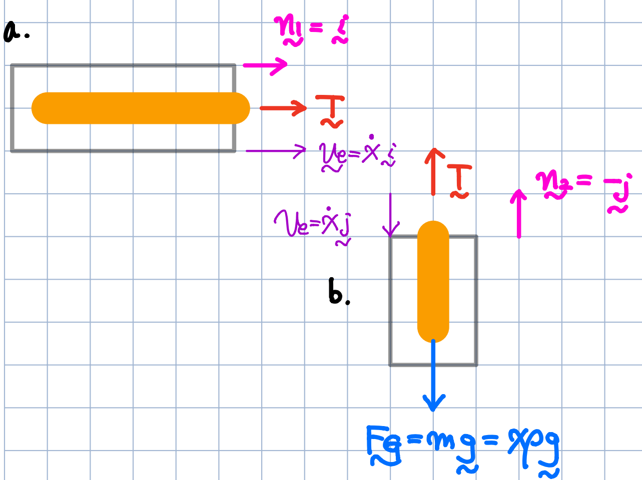
not considering rotation.

$T(x) = ?$ (tension)

1. coordinate



2. FBD



3. Reynolds Transport Theorem (EOM)

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{cv}\beta\rho\, dV\right) + \int_{cs}\beta\rho(\underset{\sim}{v}\cdot\underset{\sim}{n})\,dA$$

(a) $B = m\underset{\sim}{v}$, $\beta = \underset{\sim}{v}$

$$\frac{d}{dt}(m\underset{\sim}{v}) = \Sigma \underset{\sim}{F} = T\underset{\sim}{i} = \frac{d}{dt}\{\dot{x}\underset{\sim}{i}\,\rho(L-x)\} + (\dot{x}\underset{\sim}{i})\rho(\dot{x}\underset{\sim}{i}\cdot\underset{\sim}{i})$$

$$= \ddot{x}\rho(L-x)\underset{\sim}{i} + \cancel{\dot{x}\underset{\sim}{i}\rho(-\dot{x})} + \cancel{\dot{x}\rho\dot{x}\underset{\sim}{i}}$$

$$\rho L\ddot{x} - \rho x\ddot{x} = T \quad \cdots ①$$

(b) $B = m\underset{\sim}{v}$, $\beta = \underset{\sim}{v}$

$$\Sigma \underset{\sim}{F} = T(-\underset{\sim}{j}) + x\rho g(+\underset{\sim}{j}) = \frac{d}{dt}\{\dot{x}\underset{\sim}{j}\,\rho\, x\} + (\dot{x}\underset{\sim}{j})\rho\{\dot{x}\underset{\sim}{j}\cdot(-\underset{\sim}{j})\}$$

$$= \ddot{x}\underset{\sim}{j}\rho x + \cancel{\dot{x}\underset{\sim}{j}\rho\dot{x}} + \cancel{\dot{x}\rho\dot{x}(-\underset{\sim}{j})}$$

$$\rho x\ddot{x} = -T + x\rho g \quad \cdots ②$$

① + ②

$$\cancel{\rho}L\ddot{x} = \cancel{\rho}gx$$

$$L\ddot{x} = gx \longrightarrow \ddot{x} = \frac{gx}{L} \quad \cdots ③$$

②

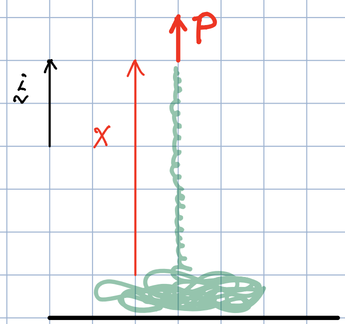
$$\rho x\left(\frac{gx}{L}\right) = -T + x\rho g$$

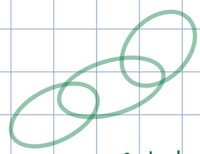
$$\therefore T = x\rho g - \rho x\left(\frac{gx}{L}\right)$$

$$= \rho g x\left(1 - \frac{x}{L}\right)$$

$$= \rho g\left(1 - \frac{x}{L}\right)x \quad \text{Ans.}$$

Example > 4.10





(open link chain)

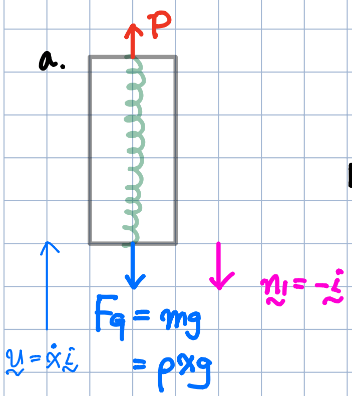
Length : L
density per unit length $= \rho$
constant v
Force : P

$P(x) = ?$

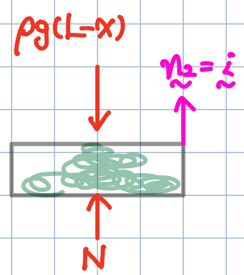
1. coordinate

2. FBD

a.



b.



3. Reynolds Transport Theorem

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{cv} \beta\rho dV\right) + \int_{cs} \beta\rho (\underset{\sim}{v}\cdot\underset{\sim}{n}) dA$$

초기의 속도=0
운동량이 0인 chain들이
들어간다.

$$(a)\quad \Sigma \underset{\sim}{F} = P\underset{\sim}{i} + \rho g x(-\underset{\sim}{i}) = \frac{d}{dt}\{(\dot{x}\underset{\sim}{i})\rho x\} + \underset{\sim}{0}\cdot\rho\cdot(\dot{x}\underset{\sim}{i}\cdot\underset{\sim}{i})$$

$$= \cancel{\ddot{x}}\underset{\sim}{i}\rho x + \dot{x}\underset{\sim}{i}\rho\dot{x}$$

$\dot{x} = \text{const}$

$$P - \rho g x = \rho\dot{x}^2 \longrightarrow P = \rho v^2 + \rho g x$$

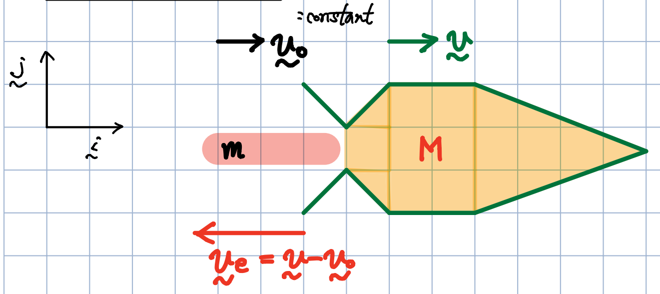
Ans.

$$(b)\quad \Sigma\underset{\sim}{F} = \rho g(L-x)(-\underset{\sim}{i}) + N\underset{\sim}{i} = \underset{\sim}{0}$$

$$\therefore N = \rho g(L-x)$$

Ans.

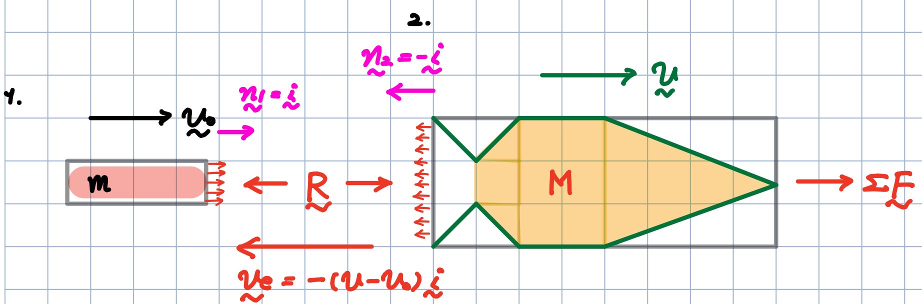
Rocket Problems



$\mathbf{u} = u\,\mathbf{i}$, $\mathbf{u}_0 = u_0\,\mathbf{i}$

$\mathbf{u}_e = (u-u_0)(-\mathbf{i})$ $(\because\ u > u_0)$

$= -(u-u_0)\,\mathbf{i}$



Reynold's Transport Theorem

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{cv} \beta\rho\, dV\right) + \int_{cs} \beta\rho(\mathbf{u}\cdot\mathbf{n})\,dA$$

1. $B = m\mathbf{u}$, $\beta = \mathbf{u}$

$d(m\mathbf{u})/dt = \Sigma\mathbf{F} = R(-\mathbf{i})$

β of CS : CS에서 순간적으로 나오는 속도 u

$= \frac{d}{dt}\left(\int_{cv} \beta_1\rho\, dV\right) + \int_{cs} \beta_1\rho(\mathbf{u}_1\cdot\mathbf{n})\,dA$

$= \frac{d}{dt}\left(\int_{cv} \mathbf{u}_0\rho\, dV\right) + \int_{cs} \mathbf{u}\rho(\mathbf{u}_e\cdot\mathbf{n}_1)\,dA$

$\mathbf{u}_0 = u_0\,\mathbf{i}$ $\quad$ $\mathbf{u} = u\,\mathbf{i}$ $\quad$ $\mathbf{u}_e = -(u-u_0)\,\mathbf{i}$

$= \dot{m}(u_0\,\mathbf{i}) + m\cancel{\dot{u}_0}\,\mathbf{i} + \rho A(u\,\mathbf{i})\cdot\{-(u-u_0)\,\mathbf{i}\cdot\mathbf{i}\}$

$u_0 = \text{const}$

$= \{\dot{m}u_0 - \rho A u(u-u_0)\}\,\mathbf{i} \leftarrow \dot{m} = \rho A(u-u_0)$

$= (\dot{m}u_0 - \dot{m}u)\,\mathbf{i}$

$= \dot{m}(u_0 - u)\,\mathbf{i}$

$\therefore R = \dot{m}(u-u_0)$

2. $B = M\mathbf{u}$, $\beta = \mathbf{u}$

$d(M\mathbf{u})/dt = \Sigma\mathbf{F} = R\,\mathbf{i} + (\Sigma F)\,\mathbf{i}$

$= \frac{d}{dt}\left(\int_{cv} \beta_2\rho\, dV\right) + \int_{cs} \beta_2\rho(\mathbf{u}_2\cdot\mathbf{n})\,dA$

$= \frac{d}{dt}\left(\int_{cv} \mathbf{u}\rho\, dV\right) + \rho A(u\,\mathbf{i})\{(u\,\mathbf{i})\cdot(-\mathbf{i})\} \leftarrow \rho A u = \dot{M}$

$= \frac{d}{dt}(M\mathbf{u}) - \dot{M}\mathbf{u}$

$= \cancel{\dot{M}\mathbf{u}} + M\dot{\mathbf{u}} - \cancel{\dot{M}\mathbf{u}}$

$= M\dot{u}\,\mathbf{i}$

$\therefore R + \Sigma F = M\dot{u}$

$\Sigma F = M\dot{u} - \dot{m}(u-u_0) \leftarrow$ $m + M = \text{const}$

$= M\dot{u} + \dot{M}(u-u_0)$ $\qquad$ $\dot{m} + \dot{M} = 0$

$\dot{m} = -\dot{M}$

추진력!