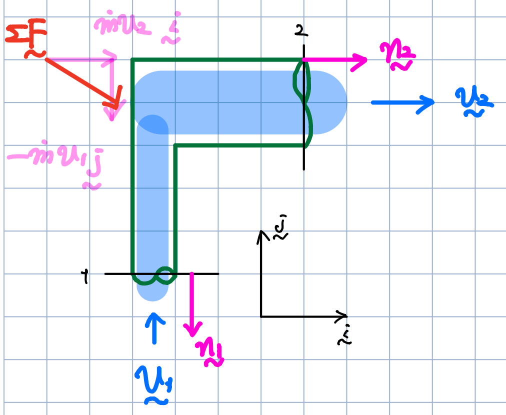
Steady Mass Flow ($\dot{m}_{in} = \dot{m}_{out}$)



$$\frac{d}{dt}(B_{syst}) = \cancel{\frac{d}{dt}\left(\int_{cv} \beta\rho \, dV\right)} + \int_{cs} \beta\rho(\mathbf{u}\cdot\mathbf{n})\,dA$$

steady-state (정상상태)

$B = m\mathbf{u}$, $\beta = \mathbf{u}$ $\quad (\beta_1 = \mathbf{u}_1,\ \beta_2 = \mathbf{u}_2)$

$$\frac{d(m\mathbf{u})}{dt} = m\mathbf{a} = \mathbf{F} = \int_{cs} \mathbf{u}\rho(\mathbf{u}\cdot\mathbf{n})\,dA$$

$= \text{surface 1} + \text{surface 2}$

$= \mathbf{u}_1\rho(\mathbf{u}_1\cdot\mathbf{n}_1)A_1 + \mathbf{u}_2\rho(\mathbf{u}_2\cdot\mathbf{n}_2)A_2 \quad \longleftarrow \quad \mathbf{u}_1 = v_1\mathbf{j}$, $\mathbf{n}_1 = -\mathbf{j}$

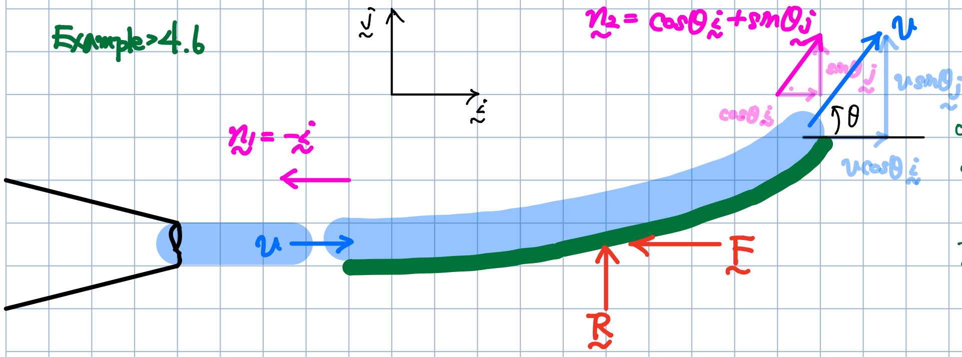
$= (v_1\mathbf{j})\rho\{v_1\mathbf{j}\cdot(-\mathbf{j})\}A_1 + (v_2\mathbf{i})\rho(v_2\mathbf{i}\cdot+\mathbf{i})A_2 \quad \mathbf{u}_2 = v_2\mathbf{i}$, $\mathbf{n}_2 = +\mathbf{i}$

$= -\rho A_1 v_1^2\mathbf{j} + \rho A_2 v_2^2\mathbf{i} \quad \longleftarrow \quad \rho A_1 v_1 = \rho A_2 v_2 = \dot{m} \quad \longleftarrow$

$$\boxed{\begin{gathered} V = uAt \\ m = \rho V = \rho u A t \\ \frac{dm}{dt} = \dot{m} = \rho u A = \rho A u \end{gathered}}$$

$= \dot{m}(v_2\mathbf{i} - v_1\mathbf{j})$ Ans.

Example 4.6



The smooth vane diverts the open stream of fluid of cross-sectional are A, mass density ρ, velocity u.

Determine $\mathbf{R}, \mathbf{F}$.

Reynolds Transport Theorem

$$\frac{d}{dt}(B_{syst}) = \cancel{\frac{d}{dt}\left(\int_{cv} \beta\rho\, dV\right)} + \int_{cs} \beta\rho(\mathbf{u}\cdot\mathbf{n})\, dA$$

$B = m\mathbf{u}$, $\beta = \mathbf{u}$

$$\frac{d}{dt}(m\mathbf{u}) = \mathbf{F} = \int_{cs} \beta\rho(\mathbf{u}\cdot\mathbf{n})\, dA$$

surface 1 : $\int_{cs} \mathbf{u}\rho(\mathbf{u}\cdot\mathbf{n})\, dA = \rho A(u\mathbf{i})\{(u\mathbf{i})\cdot(-\mathbf{i})\} = -\rho A u^2 \mathbf{i}$

surface 2 : $\int_{cs} \mathbf{u}_2\rho(\mathbf{u}_2\cdot\mathbf{n}_2)\, dA = \rho A\,(u\cos\theta\,\mathbf{i} + u\sin\theta\,\mathbf{j})\underbrace{\{(u\cos\theta\,\mathbf{i} + u\sin\theta\,\mathbf{j})\cdot(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})\}}$

$= u\cos^2\theta + u\sin^2\theta$

$= u(\sin^2\theta + \cos^2\theta)$

$= u$

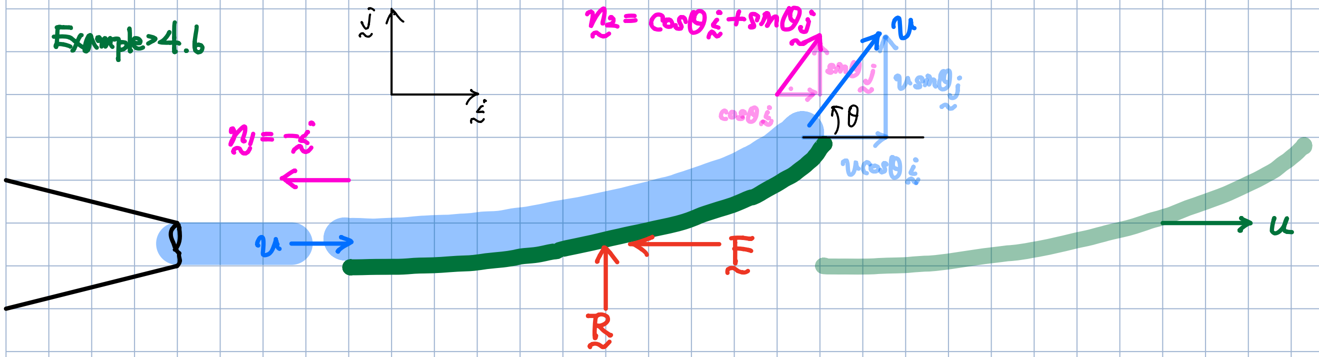
$= \rho A u^2(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$

$$\Sigma F = \mathbf{F} + \mathbf{R} = F(-\mathbf{i}) + R(\mathbf{j}) = -\rho A u^2\mathbf{i} + \rho A u^2(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

$$= \rho A u^2(-1 + \cos\theta)\,\mathbf{i} + \rho A u^2 \sin\theta\,\mathbf{j}$$

$\therefore F = \rho A u^2(1 - \cos\theta)$, $R = \rho A u^2 \sin\theta$ Ans.

Example>4.6



$$\frac{d}{dt}(B_{syst}) = \cancel{\frac{d}{dt}\left(\int_{cv}\beta\rho\,dV\right)} + \int_{cs}\beta\rho(\underset{\sim}{v}\cdot\underset{\sim}{n})\,dA$$

$B = m\underset{\sim}{v}$, $\beta = \underset{\sim}{v}$

$$\frac{d}{dt}(m\underset{\sim}{v}) = \underset{\sim}{F} = \int_{cs}\beta\rho(\underset{\sim}{v}\cdot\underset{\sim}{n})\,dA$$

surface 1 : $\int_{cs}\beta_1\rho(\underset{\sim}{v}_1\cdot\underset{\sim}{n}_1)\,dA = \rho A\,(v-u)\underset{\sim}{i}\,\{(v-u)\underset{\sim}{i}\cdot(-\underset{\sim}{i})\} = -\rho A(v-u)^2\underset{\sim}{i}$

surface 2 : $\int_{cs}\beta_2\rho(\underset{\sim}{v}_2\cdot\underset{\sim}{n}_2)\,dA = \rho A\,\{(v-u)\cos\theta\,\underset{\sim}{i} + (v-u)\sin\theta\,\underset{\sim}{j}\}\left\{\{(v-u)\cos\theta\,\underset{\sim}{i} + (v-u)\sin\theta\,\underset{\sim}{j}\}\cdot(\cos\theta\,\underset{\sim}{i} + \sin\theta\,\underset{\sim}{j})\right\}$

$$= \rho A(v-u)^2\cos\theta\,\underset{\sim}{i} + \rho A(v-u)^2\sin\theta\,\underset{\sim}{j}$$

$$\Sigma\underset{\sim}{F} = F(-\underset{\sim}{i}) + R(\underset{\sim}{j}) = \rho A(v-u)^2(\cos\theta - 1)\,\underset{\sim}{i} + \rho A(v-u)^2\sin\theta\,\underset{\sim}{j}$$

$\therefore\ F = \rho A(v-u)^2(1-\cos\theta)$, $R = \rho A(v-u)^2\sin\theta$ Ans.