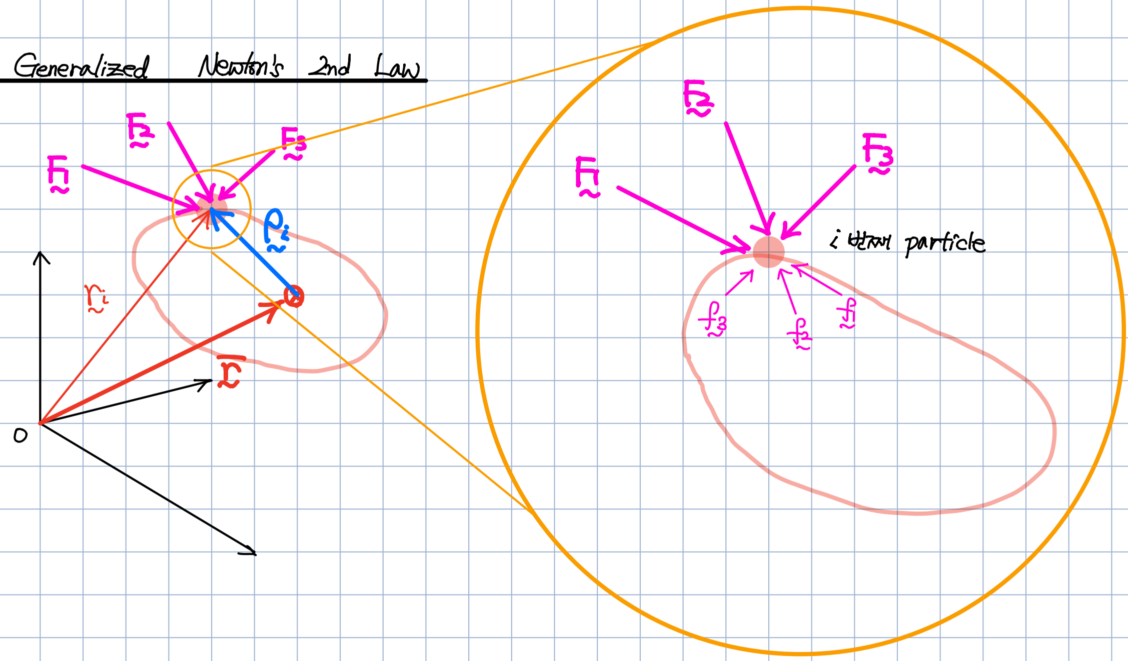
1. Generalized Newton's 2nd Law



Newton's 2nd Law : $\Sigma F = m_i \ddot{r}_i = \underbrace{F_1^{(i)} + F_2^{(i)} + F_3^{(i)}}_{\text{external forces}} + \underbrace{f_1^{(i)} + f_2^{(i)} + f_3^{(i)}}_{\text{internal forces}}$

$$
\begin{aligned}
m_1 \ddot{r}_1 &= (F_{ext})_1 + \cancel{(f_{int})_1} \\
m_2 \ddot{r}_2 &= (F_{ext})_2 + \cancel{(f_{int})_2} \\
&\vdots \\
m_i \ddot{r}_i &= (F_{ext})_i + \cancel{(f_{int})_i}
\end{aligned}
$$

$$\sum_{i=1}^{n} m_i \ddot{r}_i = \Sigma F$$

$$\Sigma m_i r_i = (\Sigma m_i)\bar{r}$$

각각의 합 = 전체의 하나

$$\Sigma m_i \underbrace{(r_i - \bar{r})}_{=\rho_i} = 0$$

$$\Sigma m_i \rho_i = 0$$

$$\Sigma m_i \ddot{r}_i = (\Sigma m_i)\ddot{\bar{r}} = \Sigma F$$

$$\boxed{\Sigma F = (\Sigma m_i)\ddot{\bar{r}}}$$

Generalized Newton's 2nd Law.

2. Work & Energy

1. one particle : $\underset{\sim}{F} \cdot d\underset{\sim}{r} = m\underset{\sim}{a} \cdot d\underset{\sim}{r} = m\underset{\sim}{v} \cdot d\underset{\sim}{v} \longleftarrow a\,ds = v\,dv$

$$U_i = \int \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int m\underset{\sim}{v} \cdot d\underset{\sim}{v} = \frac{1}{2}mv^2 = \Delta T_i$$

$U_i = \Delta T_i$ (for i-th particle)

2. Systems of particles : $\sum_i U_i = \sum \Delta T_i$

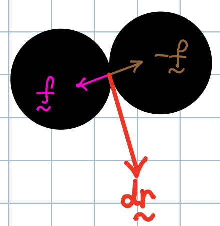
$$U = \Delta T = T_{final} - T_{initial}$$

work done

$\therefore T_{initial} + U = T_{final}$

$U = U_{external} + U_{internal}$

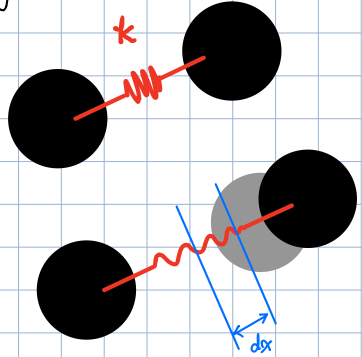
a. rigid body

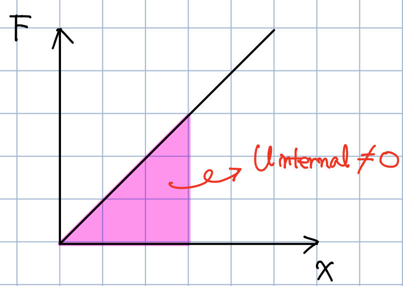


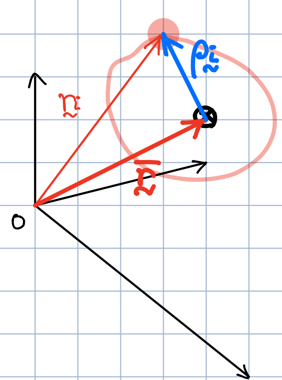
$$\underset{\sim}{f} \cdot d\underset{\sim}{r} + (-\underset{\sim}{f}) \cdot d\underset{\sim}{r} = 0$$

$\therefore U_{internal} = 0 \longrightarrow T_{initial} + U_{external} = T_{final}$

b. Nonrigid body







$$T = \sum_i^N \frac{1}{2} m_i |v_i|^2 \longleftarrow$$

$$\boxed{\begin{aligned} \underset{\sim}{r_i} &= \underset{\sim}{\bar{r}} + \underset{\sim}{\rho_i} \\ \dot{\underset{\sim}{r_i}} &= \underset{\sim}{v_i} = \dot{\underset{\sim}{\bar{r}}} + \dot{\underset{\sim}{\rho_i}} \end{aligned}}$$

$$= \sum \frac{1}{2} m_i (\dot{\underset{\sim}{\bar{r}}} + \dot{\underset{\sim}{\rho_i}})(\dot{\underset{\sim}{\bar{r}}} + \dot{\underset{\sim}{\rho_i}})$$

$$= \sum \frac{1}{2} m_i (\dot{\underset{\sim}{\bar{r}}}\dot{\underset{\sim}{\bar{r}}} + 2\dot{\underset{\sim}{\bar{r}}}\dot{\underset{\sim}{\rho_i}} + \dot{\underset{\sim}{\rho_i}}\dot{\underset{\sim}{\rho_i}})$$

$$= \sum\left(\frac{1}{2}m_i\right)\dot{\underset{\sim}{\bar{r}}}\dot{\underset{\sim}{\bar{r}}} + \cancel{\sum(m_i\dot{\underset{\sim}{\rho_i}})}\,\dot{\underset{\sim}{\bar{r}}} + \sum\left(\frac{1}{2}m_i\dot{\underset{\sim}{\rho_i}}\dot{\underset{\sim}{\rho_i}}\right) \longleftarrow$$

$$\boxed{\begin{aligned} \sum m_i \underset{\sim}{r_i} &= \left(\sum m_i\right)\underset{\sim}{\bar{r}} \\ \sum m_i (\underset{\sim}{r_i} - \underset{\sim}{\bar{r}}) &= \underset{\sim}{0} \\ \sum m_i \underset{\sim}{\rho_i} &= \underset{\sim}{0} \\ \sum m_i \dot{\underset{\sim}{\rho_i}} &= \underset{\sim}{0} \end{aligned}}$$

$$= \frac{1}{2}M|\bar{v}|^2 + \sum\left(\frac{1}{2}m_i|\dot{\underset{\sim}{\rho_i}}|^2\right)$$

CM의 운동에너지

CM에 대한 상대운동의 운동에너지.

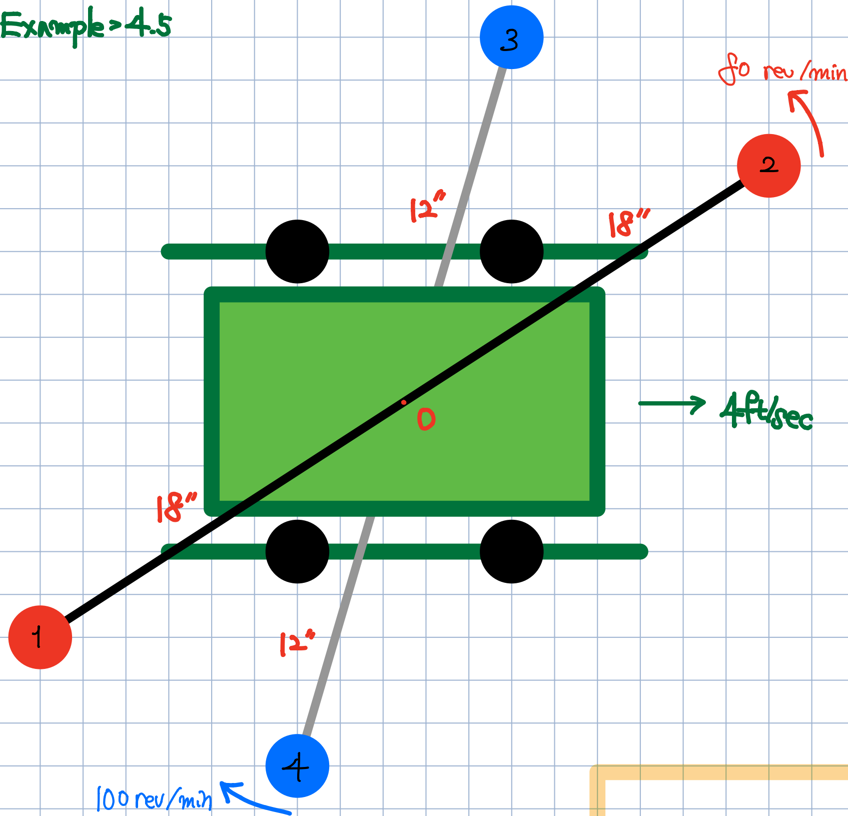
$$\boxed{T = \frac{1}{2}M|\bar{v}|^2 + \sum_{i=1}^N\left(\frac{1}{2}m_i|\dot{\rho_i}|^2\right)}$$

Example 4.5

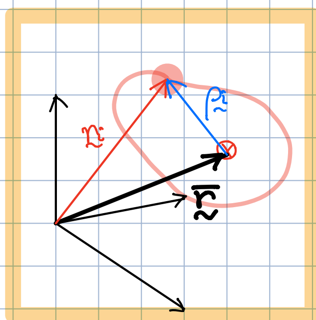
32.2 lb carriage A moves with a speed of 4 ft/sec and carries two assemblies of balls and light rods which rotate about a shaft at O.

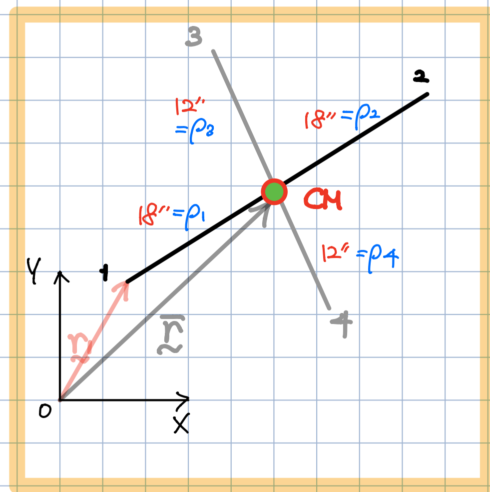
Each of the four balls weighs 3.22 lb

Determine T (Kinetic Energy)



$$\boxed{T = \frac{1}{2}M|\bar{v}|^2 + \sum_{i=1}^{N}\left(\frac{1}{2}m_i|\dot{\rho}_i|^2\right)}$$





$T = \sum \frac{1}{2}m_i|v_i|^2 \leftarrow \quad r_i = \bar{r} + \rho_i$

$\dot{r}_i = \dot{\bar{r}} + \dot{\rho}_i$

$= \sum \frac{1}{2}m_i(\dot{\bar{r}} + \dot{\rho}_i)(\dot{\bar{r}} + \dot{\rho}_i)$

$= \sum \frac{1}{2}m_i(\dot{\bar{r}}\dot{\bar{r}} + 2\dot{\bar{r}}\dot{\rho}_i + \dot{\rho}_i\dot{\rho}_i)$

$= \sum\left(\frac{1}{2}m_i\right)\dot{\bar{r}}\dot{\bar{r}} + \cancel{\sum(m_i\dot{\rho}_i)}\dot{\bar{r}} + \sum\frac{1}{2}m_i\dot{\rho}_i\dot{\rho}_i \quad \leftarrow \quad \sum m_i r_i = (\sum m_i)\bar{r}$

$\sum m_i(r_i - \bar{r}) = \sum m_i\rho_i = 0$

$\sum m_i\dot{\rho}_i = 0$

$= \underbrace{\frac{1}{2}M|\bar{v}|^2 + \sum\frac{1}{2}m_i|\dot{\rho}_i|^2}$

CM의 운동에너지 + CM에 대한 상대운동의 운동에너지

$T_{CM} = \frac{1}{2}M|\bar{v}|^2 \quad \longleftarrow \quad$ Force $= 32.2\,lb = Mg \leftarrow g = 32.2\,ft/s^2$

$M = 32.2\,lb / 32.2 = 1$

$m = 3.22\,lb / 32.2\,ft/s^2 = 0.1\,lb/(ft/s^2)$

$= \frac{1}{2}(1 + 4\times 0.1\,lb/(ft/s^2)) \cdot (4\,ft/s)^2$

$= \frac{1}{2}(1.4)\cdot 16\,\frac{lb}{ft/s^2}\cdot ft^2/s^2$

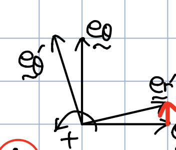
$= 11.20\,ft\text{-}lb$

$T_{rel} = \sum\frac{1}{2}m_i(\dot{\rho}_i)^2 = \frac{1}{2}m_1\dot{\rho}_1^2 + \frac{1}{2}m_2\dot{\rho}_2^2 + \frac{1}{2}m_3\dot{\rho}_3^2 + \frac{1}{2}m_4\dot{\rho}_4^2 \quad \longleftarrow$

$\rho = r_{rel}\,e_r$

$\dot{\rho} = \cancel{\dot{r}_{rel}}\,e_r + r_{rel}\,\dot{e}_r$

$\because \dot{r}_{rel} = const$



$de_r = d\theta\, e_\theta$

$\dot{e}_r = \dot{\theta}\, e_\theta$

$\dot{\rho} = r_{rel}\,\dot{\theta}\,e_\theta \quad \leftarrow$ CCW +

$\dot{\rho}_1 = \dot{\rho}_2 = (18\,in)\cdot(60\,rev/min) \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,sec} \times \frac{1\,ft}{12\,in}$

$= 12.57\,ft/s$

$\dot{\rho}_3 = \dot{\rho}_4 = (12\,in)\cdot(100\,rev/min) \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,sec} \times \frac{1\,ft}{12\,in} = 10.47\,ft/sec$

$\therefore T_{rel} = \left\{\frac{1}{2}(0.1\ lb/ft/s^2)\cdot(12.57\ ft/s)^2\right\}\times 2 + \left\{\frac{1}{2}(0.1\ lb/ft/s^2)\cdot(10.47\ ft/sec)^2\right\}\times 2$

$= 15.80 + 10.96 = 26.8\ ft\text{-}lb$

$T = T_{cu} + T_{rel}$

$= 11.20 + 26.8 = 38\ ft\text{-}lb$

Ans.