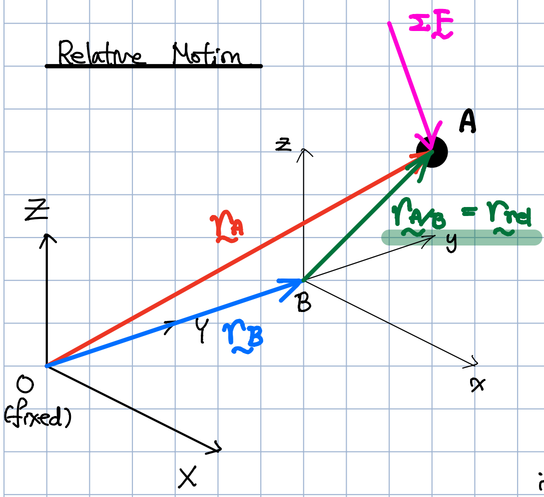
Relative Motion



$$\underset{\sim}{r}_A = \underset{\sim}{r}_{A/B} + \underset{\sim}{r}_B$$

$$\ddot{\underset{\sim}{r}}_A = \ddot{\underset{\sim}{r}}_{A/B} + \ddot{\underset{\sim}{r}}_B$$

by Newton's 2nd Law : $\Sigma \underset{\sim}{F} = m \ddot{\underset{\sim}{r}}_A$

$= m(\ddot{\underset{\sim}{r}}_{A/B} + \ddot{\underset{\sim}{r}}_B)$

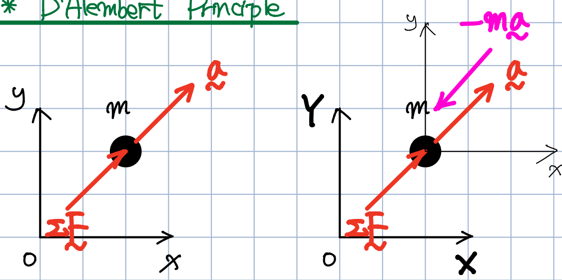
$= m(\ddot{\underset{\sim}{r}}_{rel} + \ddot{\underset{\sim}{r}}_B)$

if, $\ddot{\underset{\sim}{r}}_B \neq 0 \longrightarrow \Sigma \underset{\sim}{F} \neq m \ddot{\underset{\sim}{r}}_{rel}$

$\therefore \underset{\sim}{r}_B = \underset{\sim}{0}$, $\dot{\underset{\sim}{r}}_B = \text{const} \longrightarrow \Sigma \underset{\sim}{F} = m \ddot{\underset{\sim}{r}}_{rel}$: Newton's 2nd Law 적용가능!

↳ inertia system (= Newtonian frame of reference)

* D'Alembert Principle



$$\Sigma \underset{\sim}{F} = m \underset{\sim}{a}$$

$$\Sigma \underset{\sim}{F} - m \underset{\sim}{a} = \underset{\sim}{0}$$

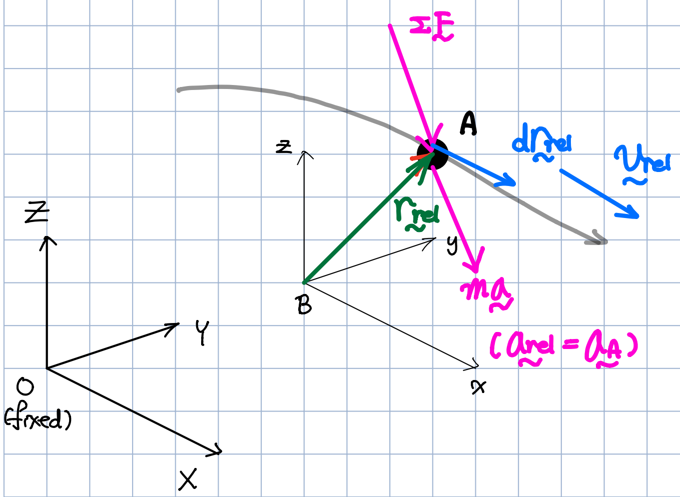
$\Sigma \underset{\sim}{F}$: actual force

$-m \underset{\sim}{a}$: inertia force

D'Alembert Principle !

1. 동적평형을 의미
2. 동역학 ⟶ 정역학.

Constant-velocity, Non-rotating System (Translating System)



$\Sigma\mathbf{F} = m(\cancel{\mathbf{a}_B} + \mathbf{a}_{rel})$

constant velocity
$(\mathbf{v}_B = \mathbf{0})$

$\boxed{dU_{rel} = \Sigma\mathbf{F}\cdot d\mathbf{r}_{rel}}$

$= m\mathbf{a}_{rel}\cdot d\mathbf{r}_{rel}$

$= m\mathbf{v}_{rel}\cdot d\mathbf{v}_{rel}$

$= d\left(\frac{1}{2}m\mathbf{v}_{rel}^2\right) \longleftarrow T = \frac{1}{2}m\mathbf{v}^2$

$a = dv/dt$, $v = ds/dt$

$dt = ds/v$

$a = \dfrac{dv}{ds/v} = v\cdot dv/ds$

$\therefore a\,ds = v\,dv$

$\therefore dU_{rel} = dT_{rel}$

$\int_A^B dU_{rel} = \int_A^B dT_{rel}$

$U_{rel} = \Delta T_{rel}$

↳ the work-energy equation holds!

$\Sigma\mathbf{F} = m\mathbf{a}_{rel}$

$\boxed{\Sigma\mathbf{F}\cdot dt = m\mathbf{a}_{rel}\cdot dt} \longleftarrow \mathbf{a}_{rel} = d\mathbf{v}_{rel}/dt$

$= m\,d\mathbf{v}_{rel}$

$= d(m\mathbf{v}_{rel})$ $(\because m = \text{const})$

$\int \Sigma\mathbf{F}\,dt = \int d(m\mathbf{v}_{rel}) = \int d\mathbf{G}_{rel}$

$\Sigma\mathbf{F} = \dot{\mathbf{G}}_{rel}$ $\int \Sigma\mathbf{F}\,dt = \Delta\mathbf{G}_{rel}$

↳ impulse-momentum equation holds

$\Sigma\mathbf{M}_B = \mathbf{r}_{rel}\times\Sigma\mathbf{F} = \mathbf{r}_{rel}\times m\mathbf{a}_{rel}$

$\Sigma\mathbf{H}_B = \mathbf{r}_{rel}\times m\mathbf{v}_{rel}$

$\dot{\mathbf{H}}_B = \frac{d}{dt}(\mathbf{r}_{rel}\times m\mathbf{v}_{rel})$

$= \cancel{\dot{\mathbf{r}}_{rel}\times m\mathbf{v}_{rel}} + \mathbf{r}_{rel}\times m\dot{\mathbf{v}}_{rel} = \mathbf{r}_{rel}\times m\mathbf{a}_{rel}$

$\dot{\mathbf{r}}_{rel} = \mathbf{v}_{rel}$

$\mathbf{v}_{rel} \parallel \mathbf{v}_{rel}$

$= \mathbf{r}_{rel}\times\Sigma\mathbf{F}$

$= \Sigma\mathbf{M}_B$

$\therefore \Sigma\mathbf{M}_B = \dot{\mathbf{H}}_B$

↳ moment-angular momentum equation holds!

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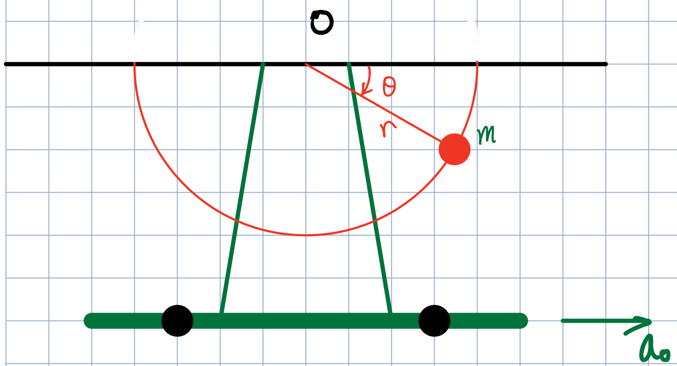
constant velocity, translating systems

1. work-energy $U_{rel} = \Delta T_{rel}$ O
2. impulse-momentum $\int\Sigma\mathbf{F}\,dt = \Delta\mathbf{G}_{rel}$ O
3. moment-angular momentum $\Sigma\mathbf{M}_B = \dot{\mathbf{H}}_B$ O
4. work $(dU = \Sigma\mathbf{F}\cdot d\mathbf{r}_A) \neq (dU_{rel} = \Sigma\mathbf{F}\cdot d\mathbf{r}_{rel})$
5. Kinetic Energy $(T = \frac{1}{2}m\mathbf{v}_A^2) \neq (T_{rel} = \frac{1}{2}m\mathbf{v}_{rel}^2)$
6. momentum $(\mathbf{G} = m\mathbf{v}) \neq (\mathbf{G}_{rel} = m\mathbf{v}_{rel})$

Example 3.32



A pendulum of mass m and length r is mounted on the flat car, which has a constant horizontal a_0.

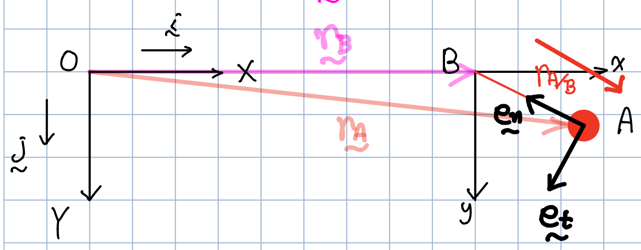
If the pendulum is released from rest to the flat car at the position $\theta = 0$,

determine the expression for the tension $T(\theta)$

" T for $\theta = \pi/2$, $\theta = \pi$

1. Coordinate

$(\ddot{r}_B = a_0)$



$r = \text{constant}$ (constraint)

2. Vectors

$$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B} \longleftarrow \underset{\sim}{r}_B = X\underset{\sim}{i}, \quad \underset{\sim}{r}_{A/B} = r(-\underset{\sim}{e}_n)$$

$$= X\underset{\sim}{i} - r\underset{\sim}{e}_n$$

$$\dot{\underset{\sim}{r}}_A = \dot{X}\underset{\sim}{i} - \cancel{\dot{r}\underset{\sim}{e}_n} - r\dot{\underset{\sim}{e}}_n$$

$\because r = \text{const}$ $\quad = -\dot{\theta}\underset{\sim}{e}_t$

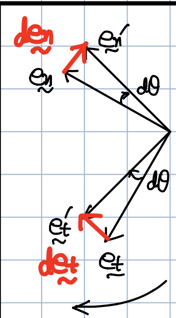
$$= \dot{X}\underset{\sim}{i} + r\dot{\theta}\underset{\sim}{e}_t$$

$$\ddot{\underset{\sim}{r}}_A = \ddot{X}\underset{\sim}{i} + \cancel{\dot{r}\dot{\theta}\underset{\sim}{e}_t} + r\ddot{\theta}\underset{\sim}{e}_t + r\dot{\theta}\dot{\underset{\sim}{e}}_t$$

$= \dot{\theta}\underset{\sim}{e}_n$

$$= \ddot{X}\underset{\sim}{i} + r\dot{\theta}^2\underset{\sim}{e}_n + r\ddot{\theta}\underset{\sim}{e}_t$$

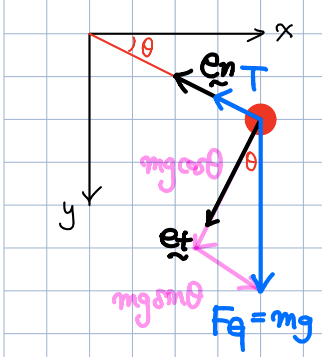
$= a_0$



$d\underset{\sim}{e}_n = d\theta \cdot (-\underset{\sim}{e}_t)$ $\quad$ $d\underset{\sim}{e}_t = d\theta \cdot (\underset{\sim}{e}_n)$

$\dot{\underset{\sim}{e}}_n = -\dot{\theta}\underset{\sim}{e}_t$ $\quad$ $\dot{\underset{\sim}{e}}_t = \dot{\theta}\underset{\sim}{e}_n$

3. FBD



$\underset{\sim}{T} = T\underset{\sim}{e}_n$

$\underset{\sim}{F}_g = mg\sin\theta(-\underset{\sim}{e}_n) + mg\cos\theta \cdot (\underset{\sim}{e}_t)$

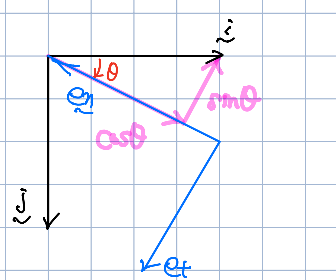
4. EOM

$$\Sigma \underset{\sim}{F} = m\underset{\sim}{a}_A$$

$$\Sigma \underset{\sim}{F} = \underset{\sim}{T} + \underset{\sim}{F}_g = (T\underset{\sim}{e}_n) + mg\sin\theta(-\underset{\sim}{e}_n) + mg\cos\theta \cdot (\underset{\sim}{e}_t)$$

$$m\underset{\sim}{a}_A = m(a_0\underset{\sim}{i} + r\dot{\theta}^2\underset{\sim}{e}_n + r\ddot{\theta}\underset{\sim}{e}_t) \longleftarrow$$

$$= m(-a_0\cos\theta\underset{\sim}{e}_n - a_0\sin\theta\underset{\sim}{e}_t + r\dot{\theta}^2\underset{\sim}{e}_n + r\ddot{\theta}\underset{\sim}{e}_t)$$



$\underset{\sim}{i} = \cos\theta(-\underset{\sim}{e}_n) + \sin\theta(-\underset{\sim}{e}_t)$

$$\boxed{\begin{aligned} T - mg\sin\theta &= m(-a_0\cos\theta + r\dot{\theta}^2) \cdots ① \\ mg\cos\theta &= m(-a_0\sin\theta + r\ddot{\theta}) \cdots ② \end{aligned}}$$

5. Calculation

① : $T = mg\sin\theta + m(-a_0\cos\theta + r\dot{\theta}^2)$

$= mg\sin\theta - ma_0\cos\theta + mr\dot{\theta}^2$

$= mg\sin\theta - ma_0\cos\theta$
$+ mr\left(\frac{2}{r}\cdot\{-a_0(\cos\theta - 1) + g\sin\theta\}\right)$

$= 2m(-a_0\cos\theta + a_0) + 2mg\sin\theta$

$= -2ma_0\cos\theta + 2ma_0 + 2mg\sin\theta$

$= -3ma_0\cos\theta + 2ma_0 + 3mg\sin\theta$

$= m\{3g\sin\theta + a_0(2 - 3\cos\theta)\}$ Ans.

② : $\cancel{m}g\cos\theta = \cancel{m}(-a_0\sin\theta + r\ddot{\theta})$

$r\ddot{\theta} = a_0\sin\theta + g\cos\theta \leftarrow d\theta$를 곱하고,

$r\ddot{\theta}d\theta = a_0\sin\theta\, d\theta + g\cos\theta\, d\theta \leftarrow a ds = v dv$

$r\dot{\theta}d\dot{\theta} = a_0\sin\theta\, d\theta + g\cos\theta\, d\theta$ $\quad \ddot{\theta}d\theta = \dot{\theta}d\dot{\theta}$

$$\int_{\dot{\theta}(0)}^{\dot{\theta}(t)} r\dot{\theta}\, d\dot{\theta} = \int_{\theta(0)}^{\theta(t)} a_0\sin\theta\, d\theta + \int_{\theta(0)}^{\theta(t)} g\cos\theta\, d\theta$$

$$\frac{1}{2}r\dot{\theta}^2\Big]_0^{\dot{\theta}} = a_0(-\cos\theta)\Big]_0^{\theta} + g\sin\theta\Big]_0^{\theta}$$

$\because \dot{\theta}(0) = 0$ rest ! $\quad = -a_0(\cos\theta - 1)$

$\frac{1}{2}r\dot{\theta}^2 = -a_0(\cos\theta - 1) + g\sin\theta$

$\dot{\theta}^2 = \frac{2}{r}\cdot\{-a_0(\cos\theta - 1) + g\sin\theta\}$

$\theta = \pi/2 \longrightarrow T_{(\theta=\pi/2)} = m\{3g\underbrace{\sin\pi/2}_{=1} + a_0(2 - \cancel{3\cos\pi/2})\}$

$= m(3g + 2a_0)$ Ans.

$\theta = \pi \longrightarrow T_{(\theta=\pi)} = m\{3g\cdot\cancel{\sin\pi} + a_0(2 - 3\underbrace{\cos\pi}_{=-1})\}$

$= 5ma_0$ Ans.