Angular

Angular Impulse & Angular Momentum

$\mathbf{M} = \mathbf{r} \times \mathbf{F}$: moment

$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$: angular momentum (O : origin)

$= (x, y, z) \times (mv_x, mv_y, mv_z)$

$$= \begin{vmatrix} i & j & k \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

$$= m(v_z y - v_y z)\,\mathbf{i} - m(v_z x - v_x z)\,\mathbf{j} + m(v_y x - v_x y)\,\mathbf{k}$$

$$\boxed{\Sigma M_O = \mathbf{r} \times \Sigma \mathbf{F} = \mathbf{r} \times m\dot{\mathbf{v}}}$$

$$\boxed{\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O}$$

$$\boxed{\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}} \longrightarrow \frac{d}{dt}\mathbf{H}_O = \frac{d}{dt}(\mathbf{r} \times m\mathbf{v})$$

$$= \dot{\mathbf{r}} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}}$$

$$= \cancel{\mathbf{v} \times m\mathbf{v}} + \mathbf{r} \times m\dot{\mathbf{v}} = \mathbf{r} \times m\dot{\mathbf{v}}$$

$\because \mathbf{v} \parallel \mathbf{v}$

$\Sigma \mathbf{M}_O = \frac{d}{dt}\mathbf{H}_O$

$\Sigma \mathbf{M}_O\, dt = d\mathbf{H}_O$

$$\boxed{\int_{t_1}^{t_2} \Sigma \mathbf{M}_O\, dt = \Delta \mathbf{H}_O}$$

각운동량의 변화량 : angular momentum

$$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O\, dt = (\mathbf{H}_O)_2$$

m 질량

$\underset{\sim}{v}$ 속도

$d\underset{\sim}{v}/dt = \underset{\sim}{a}$ 시간당 속도 변화량 (가속도)

Kinematics

$\underset{\sim}{F} = m\underset{\sim}{a}$ 힘

$\underset{\sim}{G} = m\underset{\sim}{v}$ momentum

$d\underset{\sim}{G}/dt = \dot{\underset{\sim}{G}}$ 시간당 운동량 변화량 = force

$\Delta \underset{\sim}{G}$ 운동량의 변화량 : impulse

Kinetics

$\underset{\sim}{M} = \underset{\sim}{r} \times \underset{\sim}{F}$

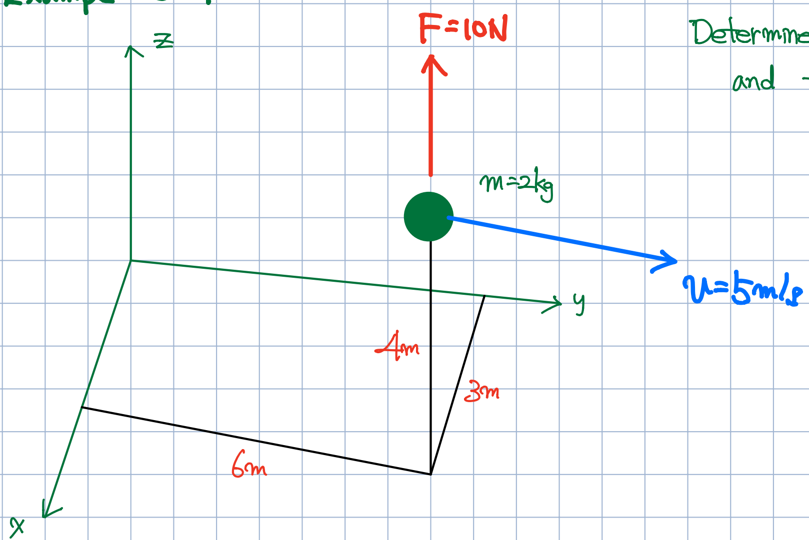
$\underset{\sim}{H_o} = \underset{\sim}{r} \times m\underset{\sim}{v}$

$\dot{\underset{\sim}{H_o}} = \Sigma \underset{\sim}{M_o}$

$\Delta \underset{\sim}{H_o} = \int_{t_1}^{t_2} \Sigma \underset{\sim}{M_o}\, dt$

$\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = m\dot{\underset{\sim}{v}} = \frac{d}{dt}(m\underset{\sim}{v})$	$\Sigma \underset{\sim}{M_o} = \underset{\sim}{r} \times \underset{\sim}{F} = \underset{\sim}{r} \times m\dot{\underset{\sim}{v}}$
$\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{G}}$	$\Sigma \underset{\sim}{H_o} = \underset{\sim}{r} \times m\underset{\sim}{v}$
$\int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\, dt = \underset{\sim}{G_2} - \underset{\sim}{G_1} = \Delta \underset{\sim}{G}$	$\Sigma \dot{\underset{\sim}{H_o}} = \dot{\underset{\sim}{r}} \times m\underset{\sim}{v} + \underset{\sim}{r} \times m\dot{\underset{\sim}{v}}$
$\boxed{\underset{\sim}{G_1} + \int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\, dt = \underset{\sim}{G_2}}$	$= \cancel{\underset{\sim}{v} \times m\underset{\sim}{v}} + \underset{\sim}{r} \times m\dot{\underset{\sim}{v}}$
	$= \Sigma \underset{\sim}{M_o}$
	$\int_{t_1}^{t_2} \Sigma M_o\, dt = \Delta \underset{\sim}{H_o}$
	$\boxed{(\underset{\sim}{H_o})_1 + \int_{t_1}^{t_2} \Sigma \underset{\sim}{M_o}\, dt = (\underset{\sim}{H_o})_2}$

Example > 3.24



Determine the angular momentum $\underset{\sim}{H}_o$ about point O and the time derivative $\dot{\underset{\sim}{H}}_o$

$\underset{\sim}{H}_o = \underset{\sim}{r} \times m\underset{\sim}{v}$ (angular momentum) $\leftarrow$ $\underset{\sim}{r} = 3\underset{\sim}{i} + 6\underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{v} = 5\underset{\sim}{j}$

$= (3\underset{\sim}{i} + 6\underset{\sim}{j} + 4\underset{\sim}{k}) \times (2)\cdot(5\underset{\sim}{j})$

$$= \begin{vmatrix} i & j & k \\ 3 & 6 & 4 \\ 0 & 10 & 0 \end{vmatrix}$$

$= -40\underset{\sim}{i} + 30\underset{\sim}{k}$ N·m/s Ans.

$\dot{\underset{\sim}{H}}_o = \frac{d}{dt}(\underset{\sim}{r} \times m\underset{\sim}{v})$

$= \dot{\underset{\sim}{r}} \times m\underset{\sim}{v} + \underset{\sim}{r} \times m\dot{\underset{\sim}{v}}$

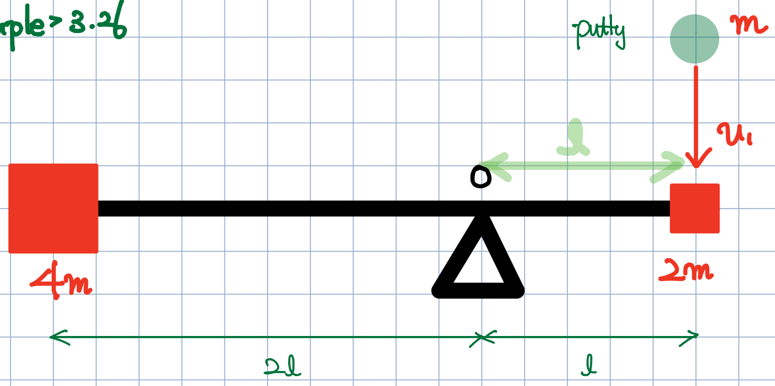
$= \cancel{\underset{\sim}{v} \times m\underset{\sim}{v}} + \underset{\sim}{r} \times m\underset{\sim}{a}$

$= \underset{\sim}{r} \times \underset{\sim}{F}$ $\leftarrow$ $\underset{\sim}{r} = 3\underset{\sim}{i} + 6\underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{F} = 10\underset{\sim}{k}$

$$= \begin{vmatrix} i & j & k \\ 3 & 6 & 4 \\ 0 & 0 & 10 \end{vmatrix}$$

$= 60\underset{\sim}{i} - 30\underset{\sim}{j}$ N·m Ans.

Example> 3.26



The assembly of the light rod and two end masses is at rest.

The putty adheres to and travels with the right-hand end mass.

Determine the angular velocity $\dot{\theta}_2$ just after the impact.

putty가 부딪히기 직전 : 1
putty가 부딪히고 난 직후 : 2

$\therefore (H_o)_1 = (H_o)_2$

↳ conservation of angular momentum.

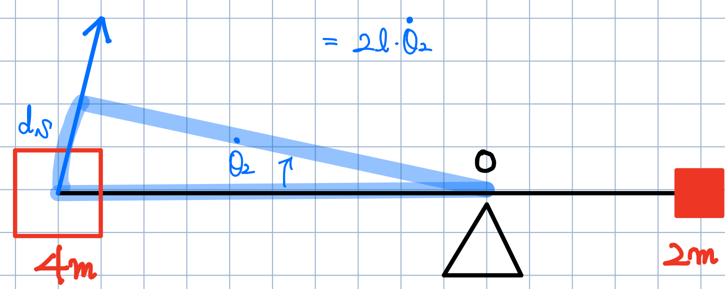
$\underset{\sim}{G} = m\underset{\sim}{v}$

$\underset{\sim}{H_o} = \underset{\sim}{r} \times m\underset{\sim}{v} = rmv\sin\theta \leftarrow \theta = 90°$

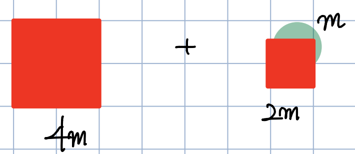
$= rmv$

$v_t = \frac{ds}{dt} = \frac{d}{dt}(2l\theta_2) = 2l \cdot \frac{d\theta_2}{dt}$

$= 2l \cdot \dot{\theta}_2$



$(H_o)_1 = rmv = (l)\cdot(m)\cdot(v_1)$

$(H_o)_2 =$ 

$= (2l)\cdot(4m)\cdot(2l\dot{\theta}_2) + (l)\cdot(3m)\cdot(l\dot{\theta}_2)$

$= 19ml^2\dot{\theta}_2$

$(H_o)_1 = (H_o)_2$

$\cancel{l}\cancel{m}v_1 = 19\cancel{m}l^{\cancel{2}}\dot{\theta}_2$

$\therefore \dot{\theta}_2 = v_1/19l$ (clockwise)

Ans.