Linear

Linear Impulse & Linear Momentum

The resultant of all forces = the time rate of change of momentum.

$$\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = m\dot{\underset{\sim}{v}} = \frac{d}{dt}(m\underset{\sim}{v}) = \dot{\underset{\sim}{G}} \quad \leftarrow \quad \underset{\sim}{G} = m\underset{\sim}{v}$$

$$\boxed{\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{G}}}$$

The linear Impulse-Momentum Principle

$$\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{G}} = \frac{d}{dt}(\underset{\sim}{G}) = \frac{d\underset{\sim}{G}}{dt}$$

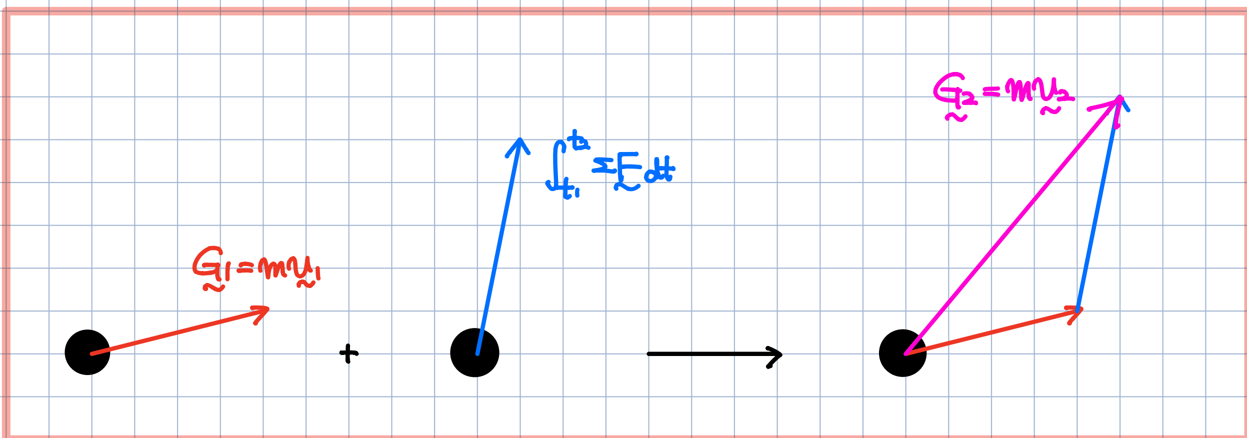
$$\Sigma \underset{\sim}{F}\, dt = d\underset{\sim}{G}$$

$$\int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\, dt = \int_{\underset{\sim}{G_1}}^{\underset{\sim}{G_2}} d\underset{\sim}{G} = \int_{\underset{\sim}{G_1}}^{\underset{\sim}{G_2}} 1\, d\underset{\sim}{G} = \underset{\sim}{G}\Big]_{\underset{\sim}{G_1}}^{\underset{\sim}{G_2}}$$

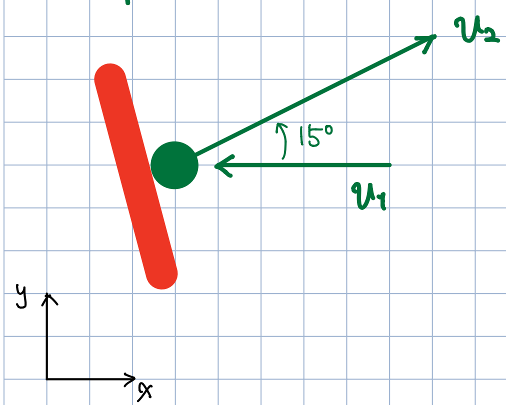
$$= \underset{\sim}{G_2} - \underset{\sim}{G_1}$$

$$\int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\, dt = \underset{\sim}{G_2} - \underset{\sim}{G_1} = \Delta \underset{\sim}{G}$$

$$\boxed{\underset{\sim}{G_1} + \int_{t_1}^{t_2} \Sigma F\, dt = \underset{\sim}{G_2}}$$



Example > 3.19



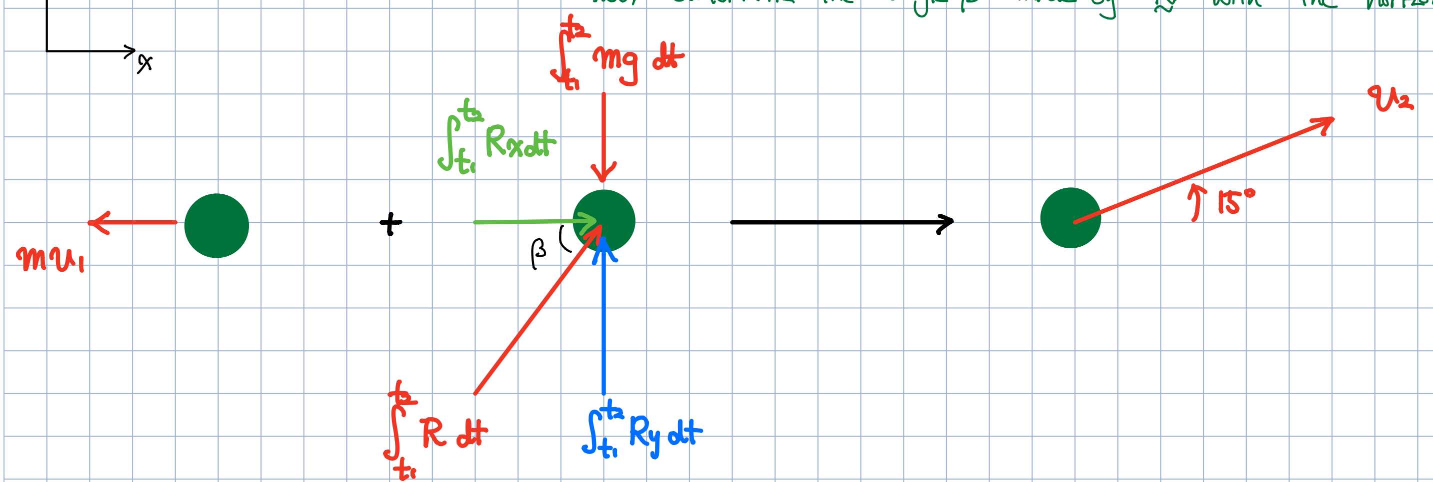
A tennis player strikes the tennis ball.

$v_1 = 50\ ft/sec$

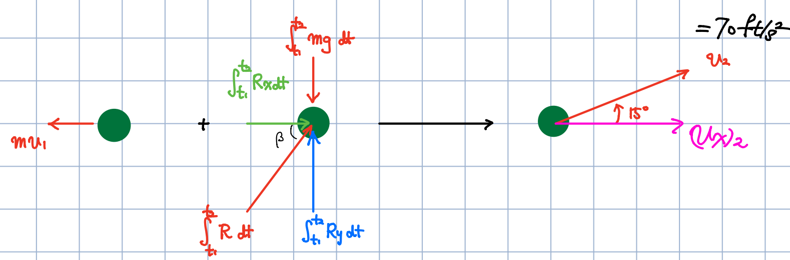
$v_2 = 70\ ft/sec$ at the 15° angle

If the 2-oz ball is in contact with the racket for 0.02 secs, determine the magnitude of the average force $\underset{\sim}{R}$ exerted by the racket.

Also, determine the angle β made by $\underset{\sim}{R}$ with the horizontal.



$$\boxed{\begin{aligned} &\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = m\dot{\underset{\sim}{v}} = \tfrac{d}{dt}(m\underset{\sim}{v}) \\ &\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{G}} \\ &\int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\,dt = \underset{\sim}{G}_2 - \underset{\sim}{G}_1 = \Delta \underset{\sim}{G} \\ &\boxed{\underset{\sim}{G}_1 + \int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\,dt = \underset{\sim}{G}_2} \end{aligned}}$$



$$\boxed{x\text{-direction}: \quad (G_x)_1 + \int_{t_1}^{t_2} F_x\,dt = (G_x)_2}$$

$$m(v_x)_1 + \int_{t_1}^{t_2} R_x\,dt = m(v_x)_2$$

$m = 2\,oz \leftarrow \quad 16\ oz = 1\ lb$

$g = 32.2\ ft/s^2$

oz, ounce : mass

lbs, pound : weight

$$\frac{2\,oz \cdot \frac{1\,lb}{16\,oz}}{32.2\ ft/s^2} \cdot (-50\ ft/s) + R_x \cdot (0.02) = \left(\frac{2 \cdot \frac{1}{16}}{32.2}\right) \cdot (70 \cos 15°)$$

$R_x = 22.8\ lb$ Ans.

$$\boxed{y\text{-direction}: \quad (G_y)_1 + \int_{t_1}^{t_2} F_y\,dt = (G_y)_2}$$

$$\cancel{\frac{2/16}{32.2} \cdot (0)} + R_y \cdot (0.02) = \frac{2/16}{32.2} \cdot (70 \sin 15°)$$

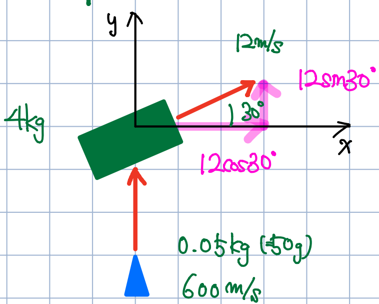
$R_y = 3.64\ lb$ Ans.

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{22.8^2 + 3.64^2} = 23.1 \text{ lb}$ Ans.

$\beta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

$= \tan^{-1}\left(\dfrac{3.64}{22.8}\right) = 9.06°$ Ans.

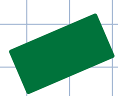
Example 13.23



If the block slides on a smooth horizontal plane with a velocity of 12m/s.

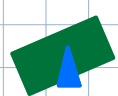
determine the velocity v_2 of the block and the embedded bullet immediately after impact.

충돌 직전 : 1 $\longrightarrow$ $G_1 = G_2$ → conservation of linear momentum.
충돌 직후 : 2

$G_1 = m v_1 =$  $+$  $= (4\text{kg})\cdot(12\cos 30° i + 12\sin 30° j) + (0.05\text{kg})\cdot(600 j)$

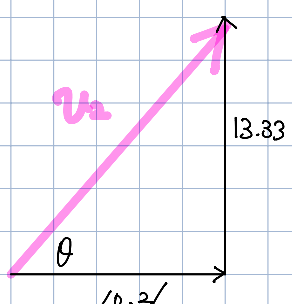
$= (41.57 i + 24 j) + 30 j$

$= 41.57 i + 54 j$

$G_2 = m v_2 =$  $= (4 + 0.05)\cdot v_2$

$= 4.05 v_2 \longrightarrow 4.05 v_2 = 41.57 i + 54 j$

$v_2 = 10.26 i + 13.33 j$ Ans



$v_2 = \sqrt{10.26^2 + 13.33^2} = 16.83 \text{ m/s}$ Ans.

$\tan\theta = v_y / v_x$

$\therefore \theta = \tan^{-1}(v_y / v_x) = 52.4°$ Ans.