Work and Energy : Kinetic Energy and Potential Energy

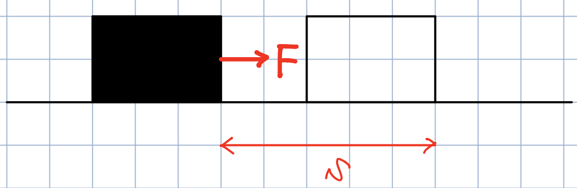
Newton's Second Law : $\underset{\sim}{F} = m\underset{\sim}{a}$

Force : The cumulative effects of unbalanced forces ?

Two general classes of problems

(1) integration of the forces with respect to the displacement.
↓
work & energy

(2) integration of the forces with respect to the time.
↓
impulse & momentum

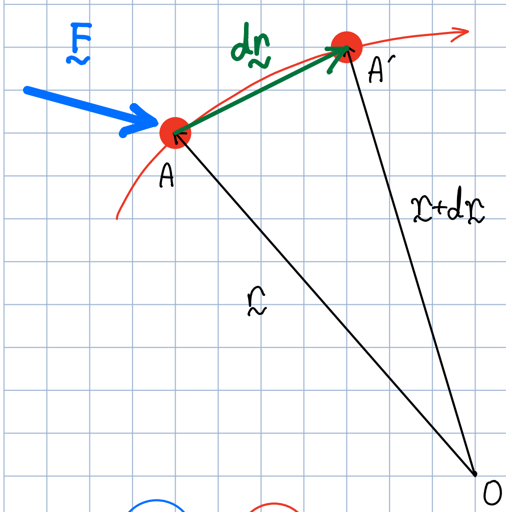


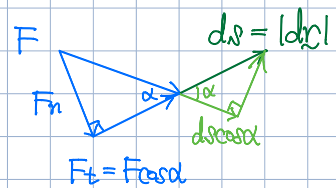
$W = Fs$: work !

Definition of Work

Work : integration of forces with respect to displacement.

$dU = \underset{\sim}{F} \cdot d\underset{\sim}{r}$ ← dot product





$dU = F\, ds\cos\alpha$

$= F\cos\alpha\, ds$

$= F_t\, ds$

$\underset{\sim}{F}$: F_n , F_t

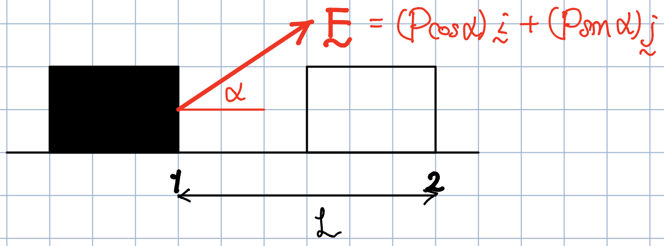
active forces : Forces which do work

reactive forces : Forces which do no work.

Examples of Work

(1) External force
(2) Spring force
(3) Weight.

(1) Work associated with a constant external force

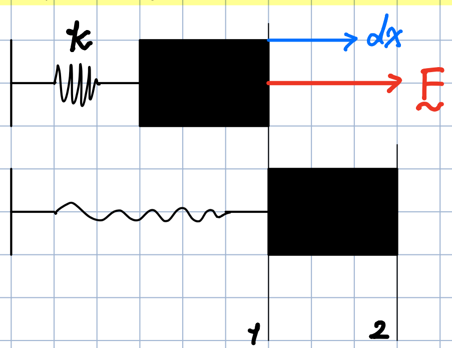


$$U_{1-2} = \int_1^2 F \cdot dr = \int_1^2 \{(P\cos\alpha)i + (P\sin\alpha)j\} \cdot dx\, i$$

$$= \int_{x_1}^{x_2} P\cos\alpha\, dx = (P\cos\alpha)\cdot(x_2 - x_1) = PL\cos\alpha$$

Ans.

(2) Work associated with a spring force



$U_{1-2} = \int_1^2 F \cdot dr$ ← F : the force exerted by the spring

$$\boxed{\begin{aligned} F &= (kx)(-i) \\ dr &= (dx)(i) \end{aligned}}$$

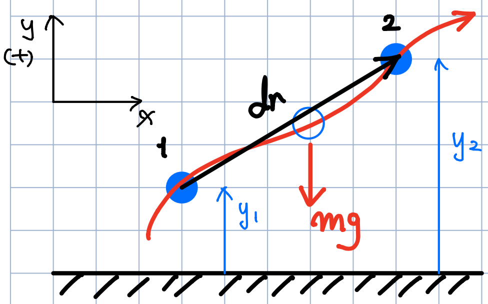
$$U_{1-2} = \int_1^2 (-kx\, i)\cdot(dx\, i) = \int_{x_1}^{x_2}(-kx)\,dx = -\int_{x_1}^{x_2} kx\, dx$$

$$= -\frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{1}{2}k(x_1^2 - x_2^2)$$

(3) Work associated with Weight

Case (a) $g = \text{constant}$



$$U_{1-2} = \int_1^2 \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

$$= \int_1^2 (mg)\cdot(-\underset{\sim}{j}) \bullet (dx\,\underset{\sim}{i} + dy\,\underset{\sim}{j})$$

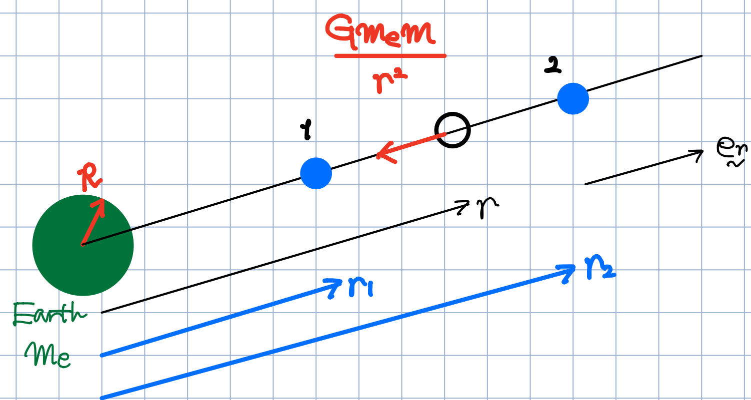
$$= \int_{y_1}^{y_2} -mg\, dy$$

$$= -mg \int_{y_1}^{y_2} dy = -mg\,(y_2 - y_1)$$

$$= mg\,(y_1 - y_2)$$

Ans.

Case (b) $g \neq \text{constant}$



Gravitation:

$$F = \frac{G m_1 m_2}{r^2}$$

F: the mutual force of attraction between two particles

G: a universal constant (= the constant of gravitation)

$$U_{1-2} = \int_1^2 \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_1^2 \left\{ \frac{G m_e m}{r^2} (-\underset{\sim}{e_r}) \right\} \cdot (dr\,\underset{\sim}{e_r})$$

$$= \int_{r_1}^{r_2} \frac{-G m_e m}{r^2}\, dr$$

$$= -G m_e m \cdot \int_{r_1}^{r_2} \frac{1}{r^2}\, dr$$

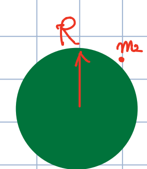
$$= -G m_e m \cdot \left. -\frac{1}{r} \right]_{r_1}^{r_2}$$

$$= -G m_e m \cdot \left(-\frac{1}{r_2} + \frac{1}{r_1}\right) \longleftarrow F = \cancel{m_2} g = G \cdot \frac{m_e \cancel{m_2}}{R^2}$$

$$= mgR^2 \cdot \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Ans.

$$G m_e = g R^2$$



Principle of Work and Kinetic Energy

Kinetic energy : $T = \frac{1}{2}mv^2$

Potential energy (work) : U_{1-2}

work-energy equation

$$\boxed{\begin{array}{l} T_1 + U_{1-2} = T_2 \\ U_{1-2} = T_2 - T_1 = \Delta T \end{array}}$$