Dynamics of Particles

1. Introduction to Dynamics : Newton's Laws , units , dimensions
2. Kinematics of Particles
3. Kinetics of Particles
4. Kinetics of Systems of Particles

Dynamics of Rigid Bodies

1. Plane Kinematics of Rigid Bodies
2. Plane Kinetics of Rigid Bodies
3. 3D Dynamics of Rigid Bodies
4. Vibration

Chapter 3. Kinetics of Particles

Force, Mass, Acceleration

Rectilinear Motion (Rectangular Coordinates)

Curvilinear Motion (Normal-Tangent , Polar Coordinates)

Work and Energy

Kinetic Energy

Potential Energy

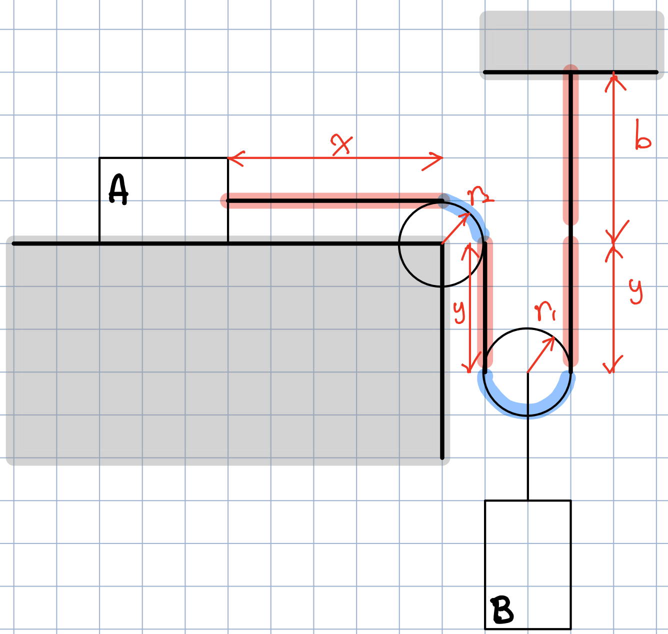
Impulse and Momentum

Linear Impulse , Linear Momentum

Angular Impulse , Angular Momentum

Constrained Motion of Connected Particles

1. One Degree of Freedom



The length of the cable :

$$L = x + \frac{2\pi r_2}{4} + 2y + \frac{2\pi r_1}{2} + b$$

$$= x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

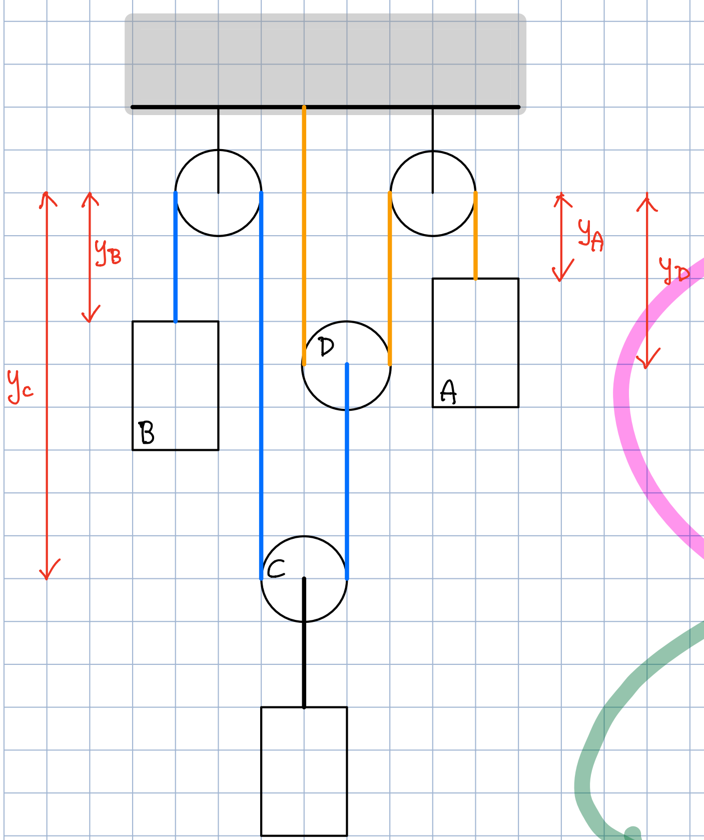
$L,\ r_2,\ r_1,\ b$ = constant

미분

$$\boxed{\begin{aligned} 0 &= \dot{x} + 2\dot{y} \\ 0 &= v_A + 2v_B \end{aligned}}$$

$$\boxed{\begin{aligned} 0 &= \ddot{x} + 2\ddot{y} \\ 0 &= a_A + 2a_B \end{aligned}}$$

2. Two Degree of Freedom



L_A : $L_A = y_A + 2y_D + \text{constant}$

$0 = \dot{y}_A + 2\dot{y}_D$

$0 = \ddot{y}_A + 2\ddot{y}_D$

L_B : $L_B = y_B + y_c + (y_c - y_D) + \text{constant}$

$0 = \dot{y}_B + 2\dot{y}_c - \dot{y}_D$

$0 = \ddot{y}_B + 2\ddot{y}_c - \ddot{y}_D$

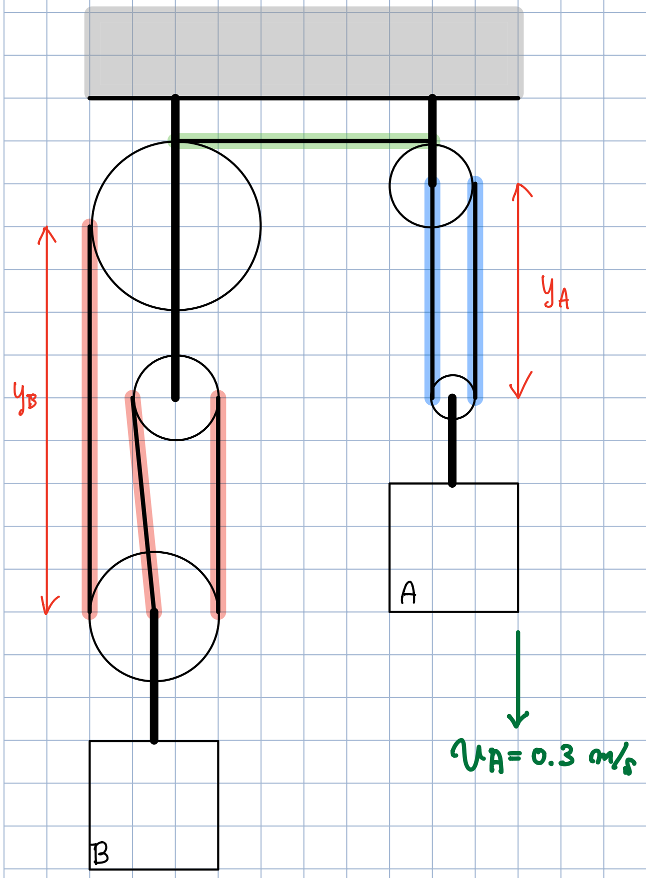
$2\dot{y}_D = 2\dot{y}_B + 4\dot{y}_c$

$0 = \dot{y}_A + 2\dot{y}_B + 4\dot{y}_c \longrightarrow \boxed{v_A + 2v_B + 4v_c = 0}$

$\ddot{y}_D = \ddot{y}_B + 2\ddot{y}_c$

$0 = \ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_c \longrightarrow \boxed{a_A + 2a_B + 4a_c = 0}$

Example: 2.15



A has a downward velocity of 0.3 m/s

Determine the velocity of B.

$$L = 2y_A + 3y_B + \text{const}$$

$$0 = 2\dot{y}_A + 3\dot{y}_B \longleftarrow \dot{y}_A = v_A = 0.3\ (\downarrow)$$

$$0 = 2v_A + 3v_B$$

$$v_B = -\tfrac{2}{3}\cdot v_A = -0.2\ m/s$$

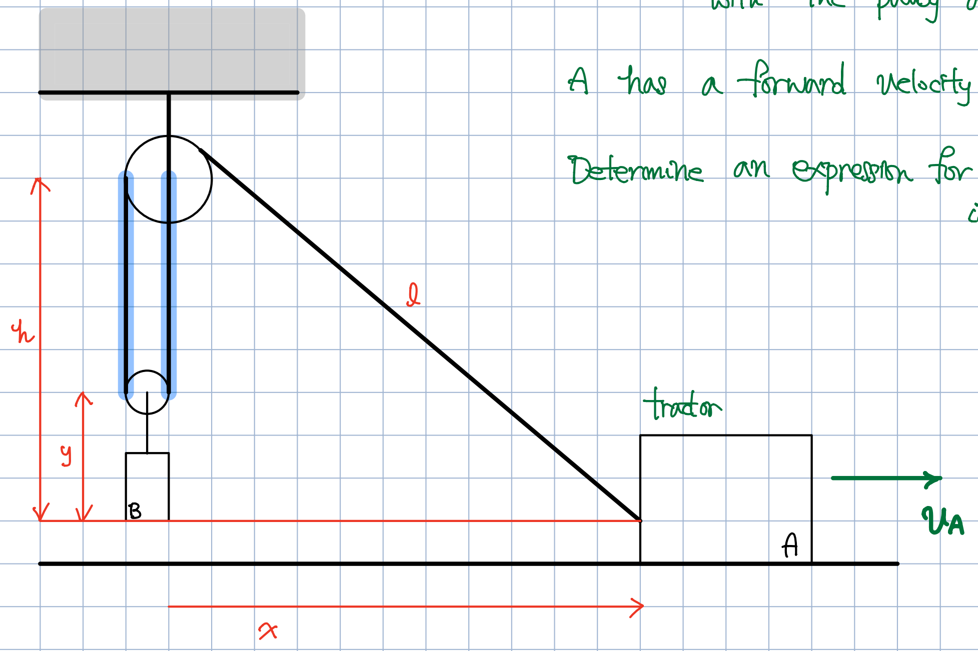
Ans.

Example > 2.16

The tractor A is used to hoist B with the pulley arrangement

A has a forward velocity v_A

Determine an expression for upward velocity v_B in terms of x



$$L = l + 2\cdot(h-y)$$

$$0 = \dot{l} - 2\dot{y} \longleftarrow l = \sqrt{x^2+h^2}, \quad \dot{l} = dl/dt = dl/dx \cdot dx/dt$$

$$dl/dx = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+h^2}}$$

$$\dot{l} = \frac{x}{\sqrt{x^2+h^2}} \cdot dx/dt$$

$$= \frac{x}{\sqrt{x^2+h^2}} \cdot \dot{x}$$

$$0 = \frac{x \cdot \dot{x}}{\sqrt{x^2+h^2}} - 2\dot{y}$$

$$v_B = \dot{y} = \frac{x \cdot v_A}{\sqrt{x^2+h^2}}$$

Ans.

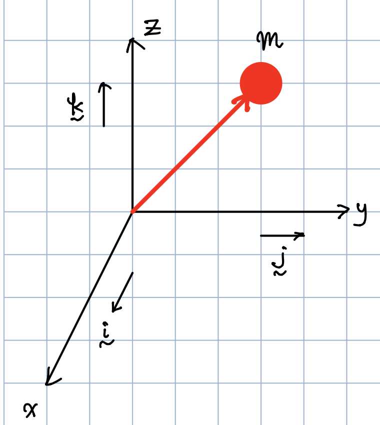
Newton's Second Law

$$\underset{\sim}{F} = m\underset{\sim}{a}$$

$$\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$$

1. Rectilinear Motion

Rectangular Coordinates



$$\underset{\sim}{a} = a_x \underset{\sim}{i} + a_y \underset{\sim}{j} + a_z \underset{\sim}{k}$$

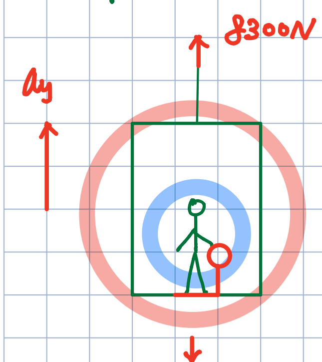
$$\Sigma F_x = m a_x$$
$$\Sigma F_y = m a_y$$
$$\Sigma F_z = m a_z$$

$$\longrightarrow \Sigma \underset{\sim}{F} = \Sigma F_x \underset{\sim}{i} + \Sigma F_y \underset{\sim}{j} + \Sigma F_z \underset{\sim}{k}$$

$$|\Sigma \underset{\sim}{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$ **Ans.**

y (+)

Example > 3.1



A 75-kg man stands on a scale in an elevator
During the first 3 secs from rest, the tension $T = 8300\,N$

The total mass of the elevator, man, scale = 750 kg

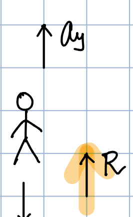
Find the reading R of the scale and the upward velocity at the end of the seconds.

$$a = \frac{dv}{dt}$$
$$dv = a\,dt$$
$$\int_0^v dv = \int_0^3 a\,dt$$
$$v - 0 = a \cdot \int_0^3 dt$$
$$= (1.257) \cdot (3)$$
$$\therefore v = 3.77\ m/s \quad \text{Ans.}$$

$F = (750) \cdot (9.81) = mg = 7360\,N$

$\Sigma F_y = m a_y \;:\; \Sigma F_y = 8300\,N - 7360\,N = m \cdot a_y$
$= (750\,kg) \cdot a_y$

$\therefore a_y = 1.257\ m/s^2$ Ans.



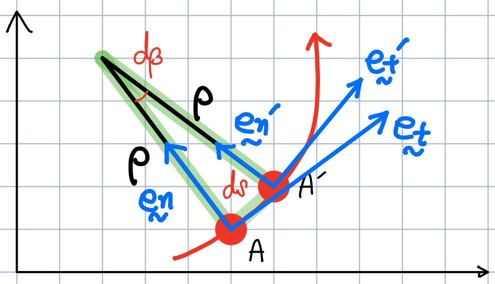
$F = mg = (75) \cdot (9.81) = 736\,N$

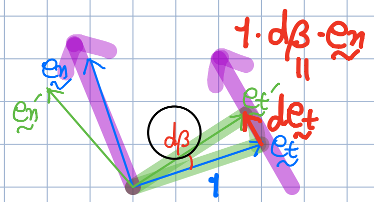
$\Sigma F_y = m a_y \;:\; \Sigma F_y = (-736) + R = m \cdot a_y$
$= (75) \cdot (1.257)$

$\therefore R = 830\,N$ Ans.

2. Curvilinear Motion

Normal and Tangential Coordinates (n-t)





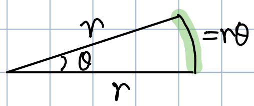
$d\mathbf{e}_t = \mathbf{e}_n \cdot d\beta \quad \leftarrow \quad$ dt로 나누면.

$\dfrac{d\mathbf{e}_t}{dt} = \mathbf{e}_n \cdot \dfrac{d\beta}{dt}$

$\dot{\mathbf{e}}_t = \dot{\beta}\,\mathbf{e}_n$

1) $\mathbf{v}$

$\boxed{\mathbf{v} = v\,\mathbf{e}_t}$



$v = \dfrac{ds}{dt} \quad \leftarrow \quad ds = \rho \cdot d\beta$

$= \rho \dfrac{d\beta}{dt}$

$= \rho\dot{\beta}$

$\boxed{\mathbf{v} = \rho\dot{\beta}\,\mathbf{e}_t}$

2) $\mathbf{a}$

$\mathbf{a} = \dfrac{d\mathbf{v}}{dt} \quad \leftarrow \quad \mathbf{v} = v\mathbf{e}_t$

$= \dfrac{d}{dt}(\mathbf{v})$

$= \dfrac{d}{dt}(v\mathbf{e}_t)$

$= \dot{v}\mathbf{e}_t + v\dot{\mathbf{e}}_t \quad \leftarrow \quad \dot{\mathbf{e}}_t = \dot{\beta}\mathbf{e}_n$

$= \dot{v}\mathbf{e}_t + v\dot{\beta}\mathbf{e}_n \quad \leftarrow \quad v = \rho\dot{\beta}, \quad \dot{\beta} = v/\rho$

$\boxed{\mathbf{a} = \dfrac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t}$

$a_n = \dfrac{v^2}{\rho} \quad \leftarrow \quad v = \rho\dot{\beta}$

$= \dfrac{\rho^2\dot{\beta}^2}{\rho}$

$= \rho\dot{\beta}^2 \quad \leftarrow \quad v = \rho\dot{\beta}$

$= v\dot{\beta}$

,

$a_t = \dot{v} = \ddot{s}$

$= \dfrac{dv}{dt} \quad \leftarrow \quad v = \rho\dot{\beta}$

$= \dfrac{d}{dt}(\rho\dot{\beta})$

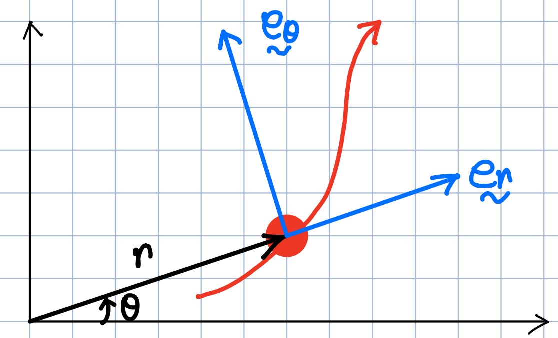
$= \dot{\rho}\dot{\beta} + \rho\ddot{\beta}$

$a = \sqrt{a_n^2 + a_t^2}$

$\therefore\ a_n = \dfrac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta} \longrightarrow \Sigma F_n = m a_n$

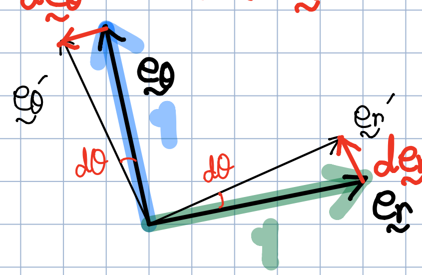
$a_t = \dot{v} = \ddot{s} = \dfrac{dv}{dt} = \dfrac{d(\rho\dot{\beta})}{dt} = \dot{\rho}\dot{\beta} + \rho\ddot{\beta} \longrightarrow \Sigma F_t = m a_t$

Polar Coordinate $(r-\theta)$



$\underset{\sim}{r} = r\underset{\sim}{e_r}$

$d\underset{\sim}{e_\theta} = 1 \cdot d\theta \cdot (-\underset{\sim}{e_r})$



$d\underset{\sim}{e_r} = 1 \cdot d\theta \cdot \underset{\sim}{e_\theta}$

$\boxed{d\underset{\sim}{e_r} = \underset{\sim}{e_\theta}\, d\theta} \longrightarrow d\underset{\sim}{e_r}/d\theta = \underset{\sim}{e_\theta}$

$\boxed{d\underset{\sim}{e_\theta} = -\underset{\sim}{e_r}\, d\theta} \longrightarrow d\underset{\sim}{e_\theta}/d\theta = -\underset{\sim}{e_r}$

$d\underset{\sim}{e_r} = \underset{\sim}{e_\theta}\, d\theta \longleftarrow$ dt로 나누기

$d\underset{\sim}{e_r}/dt = \underset{\sim}{e_\theta}\, d\theta/dt$

$\therefore \dot{\underset{\sim}{e_r}} = \underset{\sim}{e_\theta} \cdot \dot{\theta}$

$d\underset{\sim}{e_\theta} = -\underset{\sim}{e_r} \cdot d\theta$

$\dot{\underset{\sim}{e_\theta}} = -\underset{\sim}{e_r}\, \dot{\theta}$

1) $\underset{\sim}{v}$

$\underset{\sim}{r} = r\underset{\sim}{e_r} \longrightarrow \underset{\sim}{v} = \dot{\underset{\sim}{r}} = d\underset{\sim}{r}/dt$

$= d/dt\,(r\underset{\sim}{e_r})$

$= \dot{r}\underset{\sim}{e_r} + r\dot{\underset{\sim}{e_r}} \longleftarrow \dot{\underset{\sim}{e_r}} = \underset{\sim}{e_\theta} \cdot \dot{\theta}$

$\boxed{\underset{\sim}{v} = \dot{r}\underset{\sim}{e_r} + r\dot{\theta}\underset{\sim}{e_\theta}}$

2) $\underset{\sim}{a}$

$\underset{\sim}{a} = \dot{\underset{\sim}{v}} \leftarrow \underset{\sim}{v} = \dot{r}\underset{\sim}{e_r} + r\dot{\theta}\underset{\sim}{e_\theta}$

$= dv/dt$

$= \frac{d}{dt}(\dot{r}\underset{\sim}{e_r} + r\dot{\theta}\underset{\sim}{e_\theta})$

$= \ddot{r}\underset{\sim}{e_r} + \dot{r}\dot{\underset{\sim}{e_r}} + \dot{r}\dot{\theta}\underset{\sim}{e_\theta} + r\ddot{\theta}\underset{\sim}{e_\theta} + r\dot{\theta}\dot{\underset{\sim}{e_\theta}} \leftarrow \dot{\underset{\sim}{e_r}} = \underset{\sim}{e_\theta}\cdot\dot{\theta}$

$\dot{\underset{\sim}{e_\theta}} = -\underset{\sim}{e_r}\dot{\theta}$

$= \ddot{r}\underset{\sim}{e_r} + \dot{r}\dot{\theta}\underset{\sim}{e_\theta} + \dot{r}\dot{\theta}\underset{\sim}{e_\theta} + r\ddot{\theta}\underset{\sim}{e_\theta} - r\dot{\theta}^2\underset{\sim}{e_r}$

$= \boxed{(\ddot{r} - r\dot{\theta}^2)\underset{\sim}{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underset{\sim}{e_\theta}}$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \longrightarrow$ r을 곱하면, $r\cdot a_\theta = r^2\ddot{\theta} + 2\dot{r}\dot{\theta}\cdot r$

$a = \sqrt{a_r^2 + a_\theta^2}$

$= r^2\ddot{\theta} + 2r\dot{r}\dot{\theta}$

$= \frac{d}{dt}(r^2\dot{\theta})$

$\hookrightarrow 2r\dot{r}\dot{\theta} + r^2\ddot{\theta}$

$\boxed{a_\theta = \frac{1}{r}\cdot\frac{d}{dt}(r^2\dot{\theta})}$

$a_r = \ddot{r} - r\dot{\theta}^2 \longrightarrow \Sigma F_r = m\cdot a_r$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \longrightarrow \Sigma F_\theta = m a_\theta$