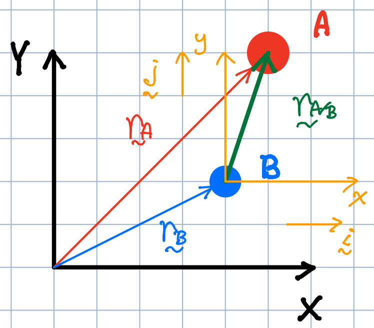
# Relative Motion

1. Choice of Coordinate system : Rectangular Coordinate

2. Vector Representation



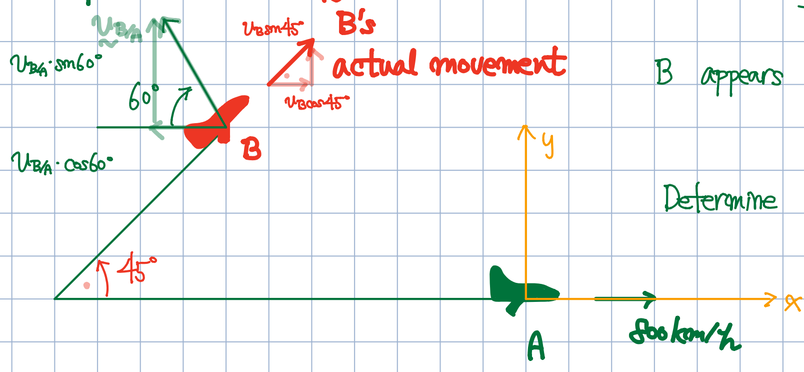

$$\underset{\sim}{r}_{A/B} = x\,\underset{\sim}{i} + y\,\underset{\sim}{j}$$

$$\boxed{\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B}}$$

$$\boxed{\begin{aligned} \dot{\underset{\sim}{r}}_A &= \dot{\underset{\sim}{r}}_B + \dot{\underset{\sim}{r}}_{A/B} \\ \underset{\sim}{v}_A &= \underset{\sim}{v}_B + \underset{\sim}{v}_{A/B} \end{aligned}}$$

$$\boxed{\begin{aligned} \ddot{\underset{\sim}{r}}_A &= \ddot{\underset{\sim}{r}}_B + \ddot{\underset{\sim}{r}}_{A/B} \\ \underset{\sim}{a}_A &= \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B} \end{aligned}}$$

## Example: 2.13

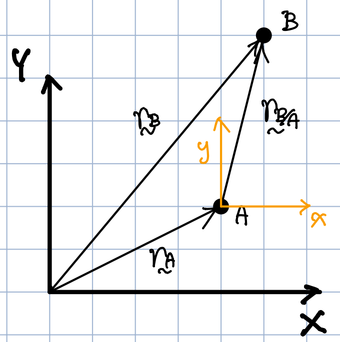



A is moving with 800 km/h in x-direction

B appears to A to be moving away from the A at the 60° angle

Determine $v_B$, $v_{BA}$ ?

observe B: 60°로 멀어지는 것처럼 보인다!




$$\underset{\sim}{r}_B = \underset{\sim}{r}_A + \underset{\sim}{r}_{BA}$$

$$\underset{\sim}{v}_B = \underset{\sim}{v}_A + \underset{\sim}{v}_{BA}$$

$$\underset{\sim}{r}_B = \underset{\sim}{r}_A + \underset{\sim}{r}_{BA}$$

$$\underset{\sim}{v}_B = \underset{\sim}{v}_A + \underset{\sim}{v}_{BA} \longleftarrow \underset{\sim}{v}_A = 800\,\underset{\sim}{i}$$

$$\underset{\sim}{v}_B = (v_B \cos 45°)\,\underset{\sim}{i} + (v_B \sin 45°)\,\underset{\sim}{j}$$

$$\underset{\sim}{v}_{BA} = (v_{BA} \cdot \cos 60°)(-\underset{\sim}{i}) + (v_{BA} \cdot \sin 60°)\,\underset{\sim}{j}$$

$$(v_B \cos 45°)\,\underset{\sim}{i} + (v_B \sin 45°)\,\underset{\sim}{j} = (800)\,\underset{\sim}{i} + (-v_{BA} \cos 60°)\,\underset{\sim}{i} + (v_{BA} \cdot \sin 60°)\,\underset{\sim}{j}$$

$$\boxed{\begin{aligned} v_B \cos 45° &= 800 - v_{BA} \cos 60° \\ v_B \sin 45° &= v_{BA} \cdot \sin 60° \end{aligned}} \longleftarrow \text{2 unknowns, 2 eqns.}$$

$$\cos 45° \cdot v_B + \cos 60° \cdot v_{BA} = 800$$

$$\sin 45° \cdot v_B - \sin 60° \cdot v_{BA} = 0$$

$$\begin{bmatrix} \cos 45° & \cos 60° \\ \sin 45° & -\sin 60° \end{bmatrix} \begin{bmatrix} v_B \\ v_{BA} \end{bmatrix} = \begin{bmatrix} 800 \\ 0 \end{bmatrix}$$

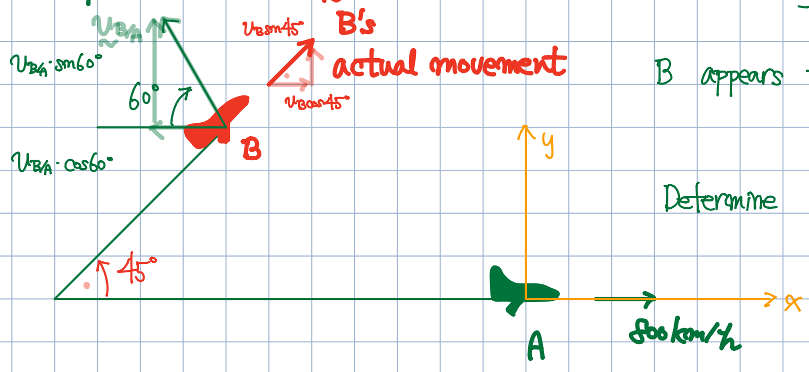
$$\begin{bmatrix} 0.7071 & 0.5 \\ 0.7071 & -0.8660 \end{bmatrix} \begin{bmatrix} v_B \\ v_{BA} \end{bmatrix} = \begin{bmatrix} 800 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_B \\ v_{BA} \end{bmatrix} = \begin{bmatrix} 0.8966 & 0.5176 \\ 0.7321 & -0.7321 \end{bmatrix} \begin{bmatrix} 800 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 717.26 \\ 585.64 \end{bmatrix}$$

Ans.

**Example: 2.13**

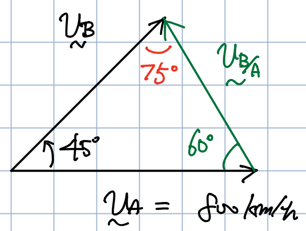



A is moving with 800 km/h in x-direction

B appears to A to be moving away from the A at the 60° angle

Determine $v_B$, $v_{B/A}$ ?

기하학적 풀이




$$\frac{v_B}{\sin 60°} = \boxed{\frac{v_{B/A}}{\sin 45°} = \frac{v_A}{\sin 75°}} \longleftarrow v_A = 800\ \text{km/h}$$

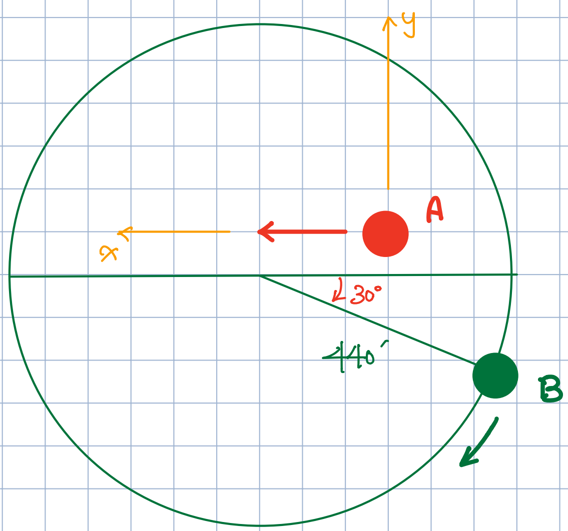
$$v_{B/A}\cdot\sin 75° = v_A\cdot\sin 45°$$

$$v_{B/A} = v_A\cdot\frac{\sin 45°}{\sin 75°} = 586\ \text{km/h}$$ Ans.

$$v_B\cdot\sin 75° = v_A\cdot\sin 60°$$

$$v_B = v_A\cdot\frac{\sin 60°}{\sin 75°} = 717\ \text{km/h}$$ Ans.

## Example > 2.14

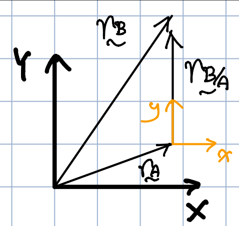



car A : accelerating at 3 ft/s²

car B : round a curve of 440 ft radius at a constant speed of 30 mi/hr

Determine the velocity and the acceleration which car B appears to have to an observer in car A if car A has reached a speed of 45 mi/hr

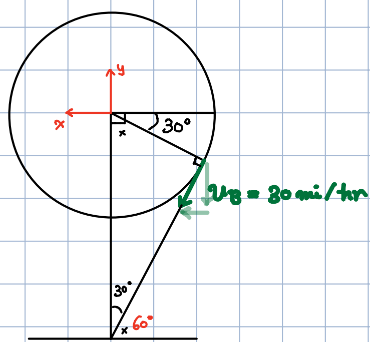
$v_{B/A}$ , $a_{B/A}$ = ?

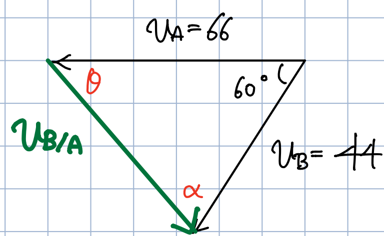



$$\underset{\sim}{r_B} = \underset{\sim}{r_A} + \underset{\sim}{r_{B/A}}$$
$$\underset{\sim}{v_B} = \underset{\sim}{v_A} + \underset{\sim}{v_{B/A}}$$
$$\underset{\sim}{a_B} = \underset{\sim}{a_A} + \underset{\sim}{a_{B/A}}$$

$$v_A = 45\ \cancel{mi}/\cancel{hr} \cdot \frac{5280\ ft}{1\ \cancel{mi}} \cdot \frac{1\ \cancel{hr}}{3600\ sec} = \underset{\sim}{66\ ft/s}$$

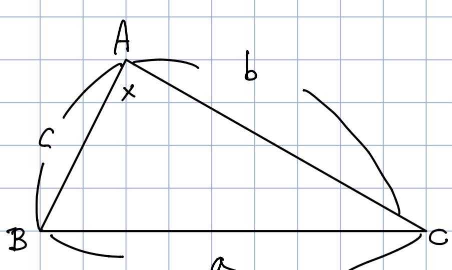
$$v_B = 30\ mi/hr \cdot \frac{5280\ ft}{1\ mi} \cdot \frac{1\ hr}{3600\ sec} = \underset{\sim}{44\ ft/s}$$




$$v_{B/A}^2 = v_A^2 + v_B^2 - 2 v_A v_B \cdot \cos 60°$$
$$= 66^2 + 44^2 - 2 \cdot 66 \cdot 44 \cdot \cos 60°$$
$$= 4356 + 1936 - 2904$$
$$= 3388$$
$$\underset{\sim}{v_{B/A} = 58.2\ ft/s}\ \text{Ans.}$$

$$\boxed{\frac{v_{B/A}}{\sin 60°} = \frac{v_B}{\sin\theta} = \frac{v_A}{\sin\alpha}}$$




$$\underline{a^2 = b^2 + c^2 - 2bc \cdot \cos A}$$

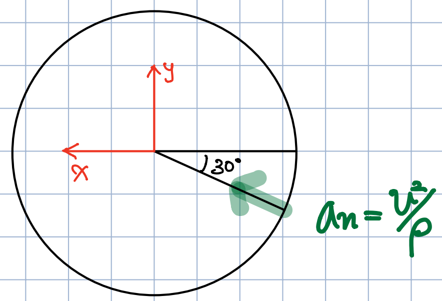
$$v_{B/A} \cdot \sin\theta = v_B \cdot \sin 60°$$

$$\sin\theta = \frac{v_B}{v_{B/A}} \cdot \sin 60°$$

$$\theta = \sin^{-1}\left(\frac{44 \cdot \sin 60°}{58.2}\right)$$

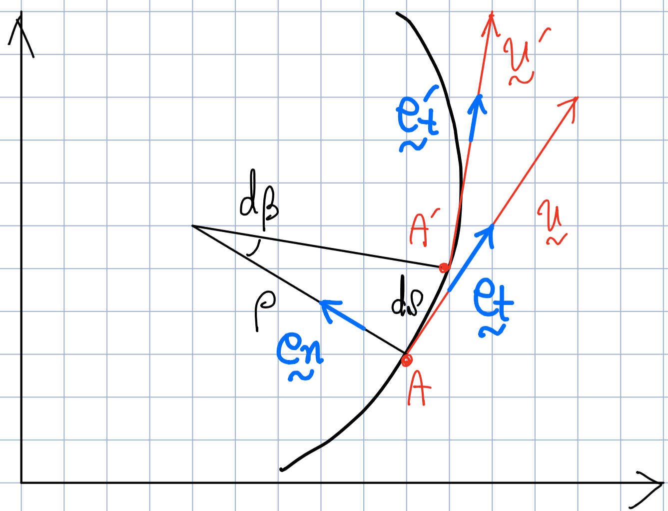
$$= \sin^{-1}(0.6547)$$

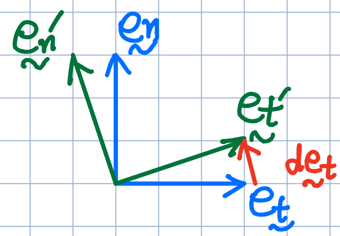
$$= \underset{\sim}{40.89°}\ \text{Ans.}$$




$v_B$ is constant $\rightarrow$ no $\underset{\sim}{e_t}$ component.

## Normal and Tangential Coordinates (n-t)




$d\underset{\sim}{e_t} = \underset{\sim}{e_n} \cdot d\beta$ ← dt로 나누면,

$d\underset{\sim}{e_t}/dt = \underset{\sim}{e_n} \cdot d\beta/dt$

$\boxed{\dot{\underset{\sim}{e_t}} = \underset{\sim}{e_n} \cdot \dot{\beta}}$

1) $\underset{\sim}{v}$

$\boxed{\underset{\sim}{v} = v\underset{\sim}{e_t}}$

$v = ds/dt$ ← $ds = \rho\, d\beta$

$= (\rho\, d\beta)/dt$

$= \rho\dot{\beta}$

$\boxed{\therefore\ \underset{\sim}{v} = \rho\dot{\beta}\,\underset{\sim}{e_t}}$

2) $\underset{\sim}{a}$

$\underset{\sim}{a} = dv/dt = v\underset{\sim}{e_t}/dt$

$\boxed{\underset{\sim}{a} = v\dot{\underset{\sim}{e_t}} + \dot{v}\underset{\sim}{e_t}}$ ← $v = \rho\beta$

$\therefore\ \underset{\sim}{a} = \rho\dot{\beta}\,\dot{\underset{\sim}{e_t}} + \dot{v}\underset{\sim}{e_t}$ ← $\dot{\underset{\sim}{e_t}} = \dot{\beta}\underset{\sim}{e_n}$

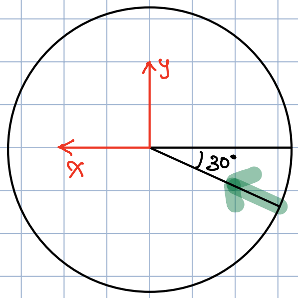
$= \rho\dot{\beta}(\dot{\beta}\underset{\sim}{e_n}) + \dot{v}\underset{\sim}{e_t}$

$= \rho\dot{\beta}^2\underset{\sim}{e_n} + \dot{v}\underset{\sim}{e_t}$ ← $v = \rho\dot{\beta}$

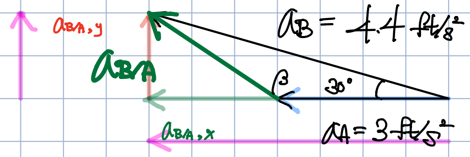
$\dot{\beta} = v/\rho$

$= \rho \cdot v^2/\rho^2\, \underset{\sim}{e_n} + \dot{v}\underset{\sim}{e_t}$

$= \boxed{v^2/\rho\, \underset{\sim}{e_n} + \cancel{\dot{v}}\,\underset{\sim}{e_t}}$




$$a_n = \frac{v^2}{\rho} = \frac{(44\ ft/s)^2}{(440\ ft)} = 4.4\ ft/s^2$$




$$a_{B/A}^2 = a_B^2 + a_A^2 - 2a_A \cdot a_B \cdot \cos 30^\circ$$

$$a_B \cdot \cos 30^\circ = a_B + a_{B/A,x} \longrightarrow a_{B/A,x} = (4.4)\cos 30^\circ - 3 = 0.8105\ ft/s^2$$

$$a_B \cdot \sin 30^\circ = a_{B/A,y} \longrightarrow a_{BA,y} = (4.4)\sin 30^\circ = 2.2\ ft/s^2$$

$$a_{B/A} = \sqrt{a_{B/A,x}^2 + a_{B/A,y}^2}$$

$$= \sqrt{0.8105^2 + 2.2^2} = 2.3445\ ft/s^2 \quad \text{Ans.}$$

$$\frac{a_B}{\sin\beta} = \frac{a_{B/A}}{\sin 30^\circ}$$

$$a_B \cdot \sin 30^\circ = a_{B/A} \cdot \sin\beta$$

$$\beta = \sin^{-1}\left(\frac{a_B \cdot \sin 30}{a_{B/A}}\right) = \sin^{-1}\left(\frac{(4.4)\cdot \sin 30^\circ}{(2.3445)}\right) = 110.2^\circ \quad \text{Ans.}$$