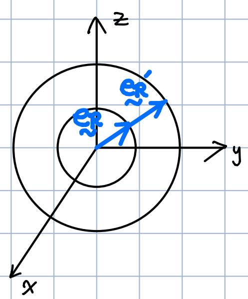
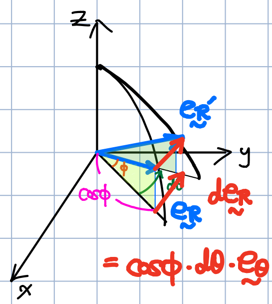
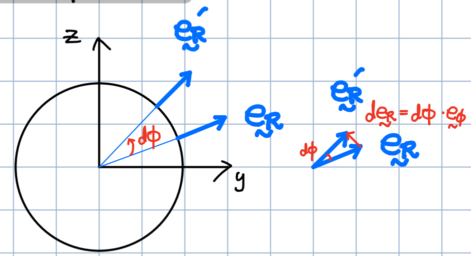
Spherical Coordinate $(R\text{-}\theta\text{-}\phi)$

$$\dot{e}_R = \frac{d e_R(R,\theta,\phi)}{dt} = \frac{\partial e_R}{\partial R}\cdot\dot{R} + \frac{\partial e_R}{\partial \theta}\cdot\dot{\theta} + \frac{\partial e_R}{\partial \phi}\dot{\phi}$$

$\boxed{\partial e_R/\partial R = 0}$ $\boxed{\partial e_R/\partial \theta = \cos\phi\, e_\theta}$ $\boxed{\partial e_R/\partial \phi = e_\phi}$

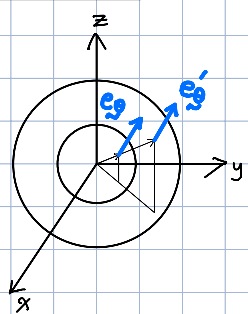


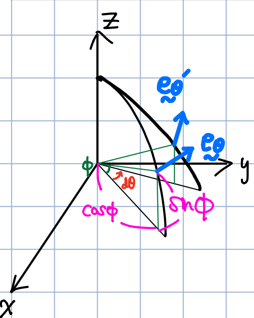




$$\dot{e}_\theta = \frac{d e_\theta(R,\theta,\phi)}{dt} = \frac{\partial e_\theta}{\partial R}\cdot\dot{R} + \frac{\partial e_\theta}{\partial \theta}\dot{\theta} + \frac{\partial e_\theta}{\partial \phi}\dot{\phi}$$

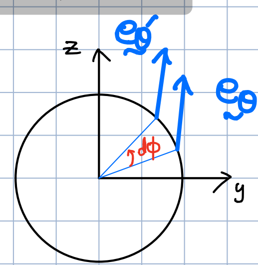
$\boxed{\partial e_\theta/\partial R = 0}$ $\boxed{\partial e_\theta/\partial \theta = -\cos\phi\, e_R + \sin\phi\, e_\phi}$ $\boxed{\partial e_\theta/\partial \phi = 0}$

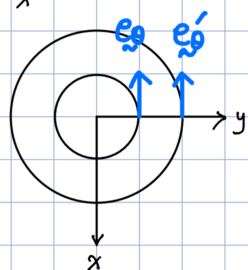




$de_\theta = \cos\phi \cdot (-e_R)$

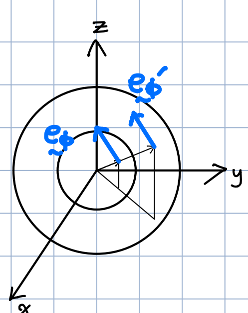
$de_\theta = \sin\phi \cdot (e_\phi)$

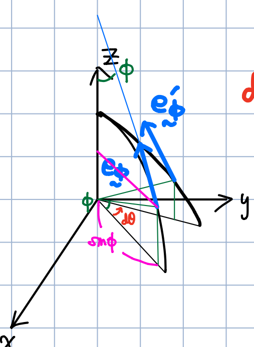




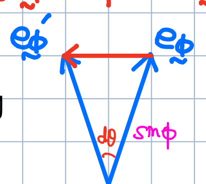
$$\dot{e}_\phi = \frac{d e_\phi(R,\theta,\phi)}{dt} = \frac{\partial e_\phi}{\partial R}\cdot\dot{R} + \frac{\partial e_\phi}{\partial \theta}\dot{\theta} + \frac{\partial e_\phi}{\partial \phi}\dot{\phi}$$

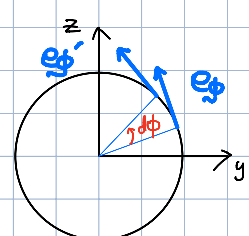
$\boxed{\partial e_\phi/\partial R = 0}$ $\boxed{\partial e_\phi/\partial \theta = -\sin\phi\, e_\theta}$ $\boxed{\partial e_\phi/\partial \phi = -e_R}$



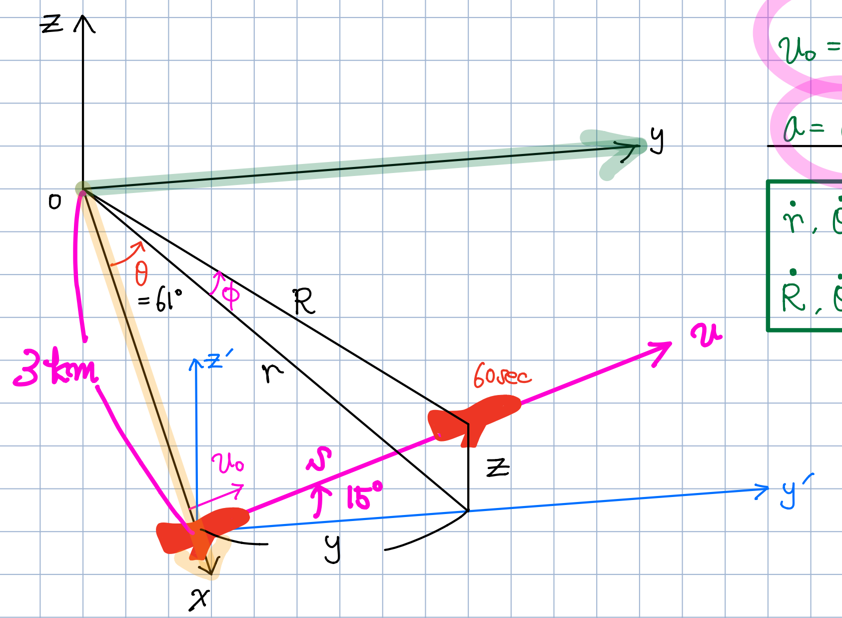


$de_\phi = \sin\phi \cdot d\theta \cdot (-e_\theta)$





Example > 2.12 (cylindrical & spherical coordinate)



$u_0 = 250\ km/h = 250 \dfrac{\cancel{km}}{\cancel{h}} \cdot \dfrac{1000\ m}{1\ \cancel{km}} \cdot \dfrac{1\ \cancel{h}}{3600\ s}$

$a = 0.8\ m/s^2 \qquad = 69.4\ m/s$

$\boxed{\begin{matrix} \dot r, \dot\theta, \dot z \\ \dot R, \dot\theta, \dot\phi \end{matrix}}$ (after 60s) ?

$$\boxed{\begin{aligned} &\text{Cylindrical coordinate:} && \underset{\sim}{R} = r\underset{\sim}{e_r} + z\underset{\sim}{k} \\ & && \underset{\sim}{v} = \dot r\underset{\sim}{e_r} + r\dot\theta\underset{\sim}{e_\theta} + \dot z\underset{\sim}{k} \\ &\text{Spherical coordinate:} && \underset{\sim}{R} = R\underset{\sim}{e_R} \\ & && \underset{\sim}{v} = \dot R\underset{\sim}{e_R} + R\dot\theta\cos\phi\underset{\sim}{e_\theta} + R\dot\phi\underset{\sim}{e_\phi} \end{aligned}}$$

$a = \dfrac{du}{dt}$

$du = a\,dt$

$\int_{u_0}^{u} du = \int_0^t a\,dt = a \cdot \int_0^t dt$

$u - u_0 = a \cdot t$

$\therefore u = u_0 + at$

$u = \dfrac{ds}{dt}$

$ds = u\,dt$

$\int_0^s ds = \int_0^t u\,dt \leftarrow u = u_0 + at$

$s = u_0 t + \frac{1}{2}at^2$

$s = u_0 t + \frac{1}{2}at^2$

$= (69.4\ m/s) \cdot (60\ sec) + \frac{1}{2} \cdot (0.8\ m/s^2) \cdot (60\ sec)^2 = 5610\ m$

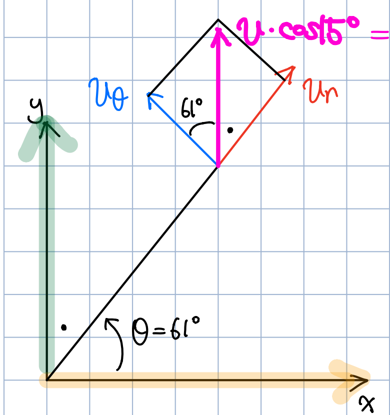
$y = s \cdot \cos 15° = (5610\ m) \cdot (\cos 15°) = 5420\ m$

$r = \sqrt{(3000\ m)^2 + y^2} = \sqrt{(3000\ m)^2 + (5420\ m)^2} = 6190\ m$

$\tan\theta = \dfrac{y}{3000} \longrightarrow \theta = \tan^{-1}(y/3000) = \tan^{-1}(5420/3000) = 61°$

$u = u_0 + at = (69.4\ m/s) + (0.8\ m/s^2) \cdot (60\ sec) = 117.4\ m/s$

$u \cdot \cos 15° = (117.4\ m/s) \cdot (\cos 15°) = 113.4\ m/s$



$u_\theta = (u \cdot \cos 15°) \cdot (\cos 61°) = (113.4\ m/s) \cdot (\cos 61°) = 55\ m/s$

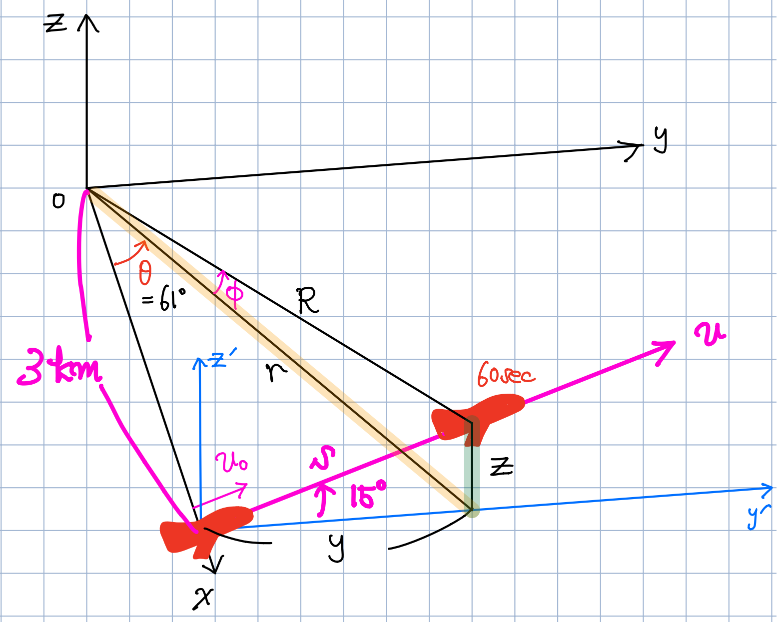
$u_\theta = r\dot\theta \longrightarrow \dot\theta = u_\theta / r = \dfrac{55\ m/s}{6190\ m} = 8.88 \times 10^{-3}\ rad/s$ Ans

$u_r = (u \cdot \cos 15°) \cdot (\sin 61°) = (113.4\ m/s) \cdot (\sin 61°) = 99.2\ m/s$

$u_r = \dot r = 99.2\ m/s$ Ans

$v_z = v \cdot \sin 15° = (117.4\,m/s) \cdot (\sin 15°) = 30.4\,m/s$

$v_z = \dot{z} = 30.4\,m/s$ Ans



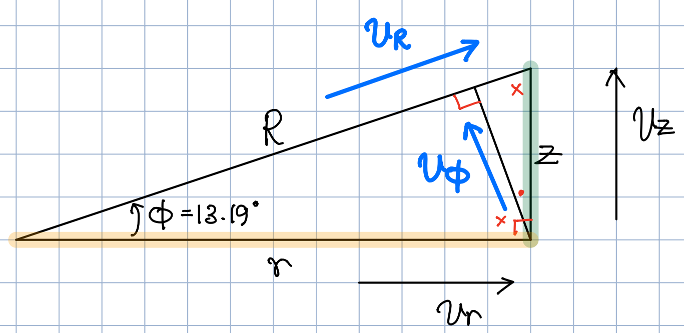
$\tan\phi = z/r$

$\phi = \tan^{-1}(z/r) = \tan^{-1}\left(\dfrac{1451\,m}{6190\,m}\right) = 13.19°$

$z = \tan 15° \cdot y$

$= (5420\,m) \cdot (\tan 15°) = 1451\,m$

$R = \sqrt{r^2+z^2} = \sqrt{6190^2 + 1451^2} = 6360\,m$



$v_R = v_r \cdot (\cos 13.19°) + v_z \cdot (\sin 13.19°)$

$= (99.2\,m/s) \cdot (\cos 13.19°) + (30.4\,m/s) \cdot (\sin 13.19°)$

$= 103.6\,m/s$

$v_R = \dot{R} = 103.6\,m/s$ Ans

$v_\phi = -v_r \cdot (\sin 13.19°) + v_z \cdot (\cos 13.19°)$

$= (-99.2\,m/s) \cdot (\sin 13.19°) + (30.4\,m/s) \cdot (\cos 13.19°)$

$= 6.95\,m/s$

$v_\phi = R\dot{\phi} \longrightarrow \dot{\phi} = v_\phi / R = \dfrac{6.95\,m/s}{6360\,m} = 1.093 \times 10^{-3}\,rad/s$ Ans