[동역학 8강]

8강 17분 10초 라운드e세타/라운드t 에서 sin성분이 왜 있는지를 모르겠으며 핑크색 박스에서 e세타의 길이가

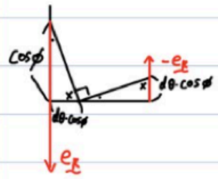
왜 sin세타인지 잘 이해가 안가네요ㅠㅠ 제가 생각했을때 세타 방향으로 돌려도 xy평명을 기준으로 e세타와 e세타 프라임의 높이차가 없다고 생각했거든요. 그래서 그런지 sin성분이 왜 있는지를 이해가 안가네요.

ii) $\frac{\partial e_\theta}{\partial \theta}$

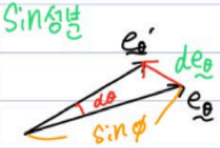
Cos성분

$de_\theta = \cos\phi \, d\theta \, (-e_r)$

$\frac{de_\theta}{d\theta} = \cos\phi \, (-e_r)$



Sin성분



$de_\theta = d\theta \sin\phi \cdot e_\phi$

$\frac{de_\theta}{d\theta} = \sin\phi \cdot e_\phi$

(17분 10초)

e_θ의 길이가 왜 $\sin\phi$ 이해x

$\frac{de_\theta}{d\theta} = -\cos\phi \, e_r + \sin \, e_\phi$

Q 동역학 8강 αeθ/αθ구하는 과정 2020-03-25 오후 4:08:00 답변수 : 1

[강좌명] 동역학 한방에 끝내기

안녕하세요 교수님!

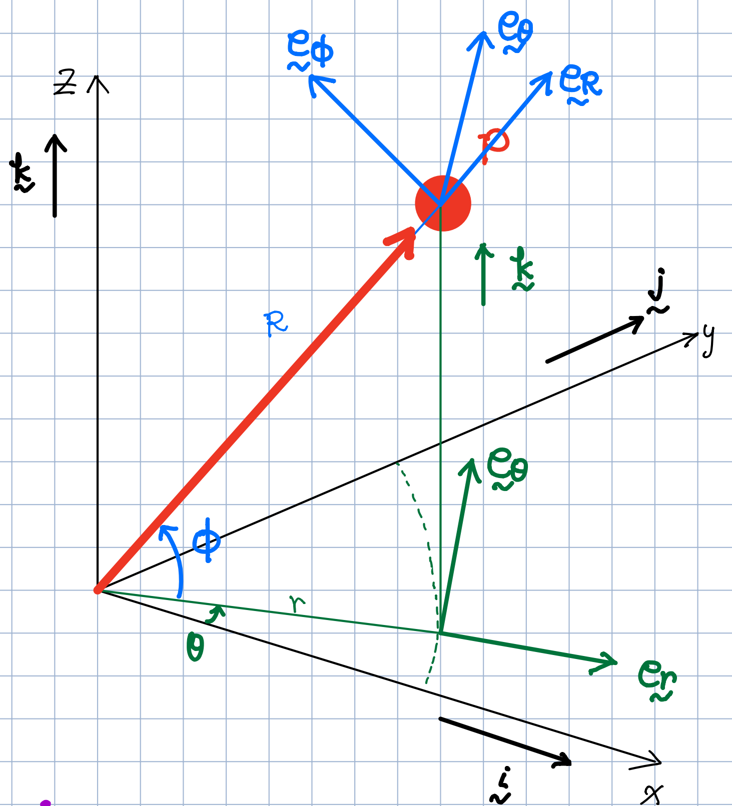
αeθ/αθ을 구하는 과정에서 sin성분에대한 이해가 잘 되지 않습니다..ㅠ

여기선 θ가 변할때에 관한 과정이므로 Φ에대한 변화가 생기지 않는다고 생각이 되어져서 sin성분의 변화가 잘 그려지지 않습니다..

혹시 더 자세한 설명이 가능하시면 부탁드리겠습니다!

감사합니다.

3. Spherical Coordinate (R-θ-φ)



$$\boxed{\mathbf{R} = R\,\mathbf{e}_R}$$

$$\mathbf{v} = \dot{\mathbf{R}} = \dot{R}\,\mathbf{e}_R + R\,\dot{\mathbf{e}}_R$$

$$\dot{\mathbf{e}}_R = \frac{d\,\mathbf{e}_R(R,\theta,\phi)}{dt} = \cos\phi\cdot\dot{\theta}\,\mathbf{e}_\theta + \dot{\phi}\,\mathbf{e}_\phi$$

$$= \frac{1}{dt}\left\{\frac{\partial \mathbf{e}_R}{\partial R}\cdot dR + \frac{\partial \mathbf{e}_R}{\partial \theta}\cdot d\theta + \frac{\partial \mathbf{e}_R}{\partial \phi}\cdot d\phi\right\}$$

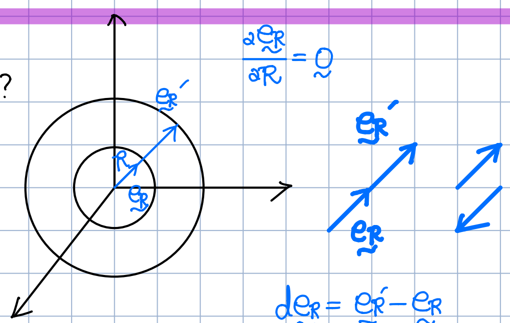
$$= \underbrace{\frac{\partial \mathbf{e}_R}{\partial R}}_{=0}\cdot\dot{R} + \underbrace{\frac{\partial \mathbf{e}_R}{\partial \theta}}_{=\cos\phi\cdot\mathbf{e}_\theta}\cdot\dot{\theta} + \underbrace{\frac{\partial \mathbf{e}_R}{\partial \phi}}_{=\mathbf{e}_\phi}\cdot\dot{\phi}$$

$$\mathbf{v} = \dot{R}\,\mathbf{e}_R + R\,(\cos\phi\cdot\dot{\theta}\,\mathbf{e}_\theta + \dot{\phi}\,\mathbf{e}_\phi)$$

$$= \boxed{\dot{R}\,\mathbf{e}_R + R\dot{\theta}\cos\phi\,\mathbf{e}_\theta + R\dot{\phi}\,\mathbf{e}_\phi}$$

$\dot{\mathbf{e}}_R$

i. $\partial \mathbf{e}_R / \partial R$?



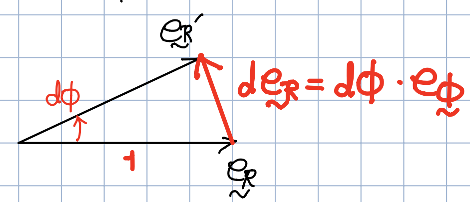
$$\frac{\partial \mathbf{e}_R}{\partial R} = 0$$

$$d\mathbf{e}_R = \mathbf{e}_R' - \mathbf{e}_R$$

$$= \nearrow \swarrow$$

$$= 0$$

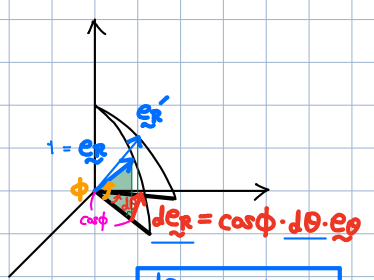
iii. $\partial \mathbf{e}_R / \partial \phi$



$$d\mathbf{e}_R = d\phi\cdot\mathbf{e}_\phi$$

$$\boxed{d\mathbf{e}_R / d\phi = \mathbf{e}_\phi}$$

ii. $\partial \mathbf{e}_R / \partial \theta$



$$d\mathbf{e}_R = \cos\phi\cdot d\theta\cdot\mathbf{e}_\theta$$

$$\boxed{d\mathbf{e}_R / d\theta = \cos\phi\cdot\mathbf{e}_\theta}$$

$$\boxed{\mathbf{v} = \dot{R}\,\mathbf{e}_R + R\dot{\theta}\cos\phi\,\mathbf{e}_\theta + R\dot{\phi}\,\mathbf{e}_\phi}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{R}\,\mathbf{e}_R + \dot{R}\,\dot{\mathbf{e}}_R + \qquad = \dot{\theta}\cos\phi\,\mathbf{e}_\theta + \dot{\phi}\,\mathbf{e}_\phi$$

$$\dot{R}\dot{\theta}\cos\phi\,\mathbf{e}_\theta + R\ddot{\theta}\cos\phi\,\mathbf{e}_\theta + R\dot{\theta}(-\sin\phi)\cdot\dot{\phi}\cdot\mathbf{e}_\theta + R\dot{\theta}\cos\phi\,\dot{\mathbf{e}}_\theta +$$

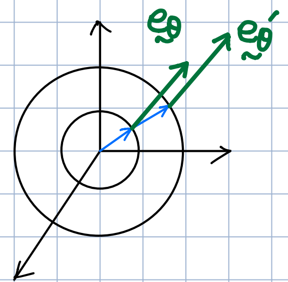
$$\dot{R}\dot{\phi}\,\mathbf{e}_\phi + R\ddot{\phi}\,\mathbf{e}_\phi + R\dot{\phi}\,\dot{\mathbf{e}}_\phi$$

$$\dot{\underset{\sim}{e}}_\theta = \frac{d\underset{\sim}{e}_\theta(R,\theta,\phi)}{dt} = \frac{1}{dt}\cdot\left\{\frac{\partial e_\theta}{\partial R}\cdot dR + \frac{\partial e_\theta}{\partial \theta}\cdot d\theta + \frac{\partial e_\theta}{\partial \phi}d\phi\right\}$$

$$= \underbrace{\cancel{\frac{\partial e_\theta}{\partial R}}}_{=0}\cdot \dot R + \underbrace{\frac{\partial e_\theta}{\partial \theta}}_{=-\cos\phi\cdot \underset{\sim}{e}_R + \sin\phi\, \underset{\sim}{e}_\phi}\dot\theta + \cancel{\frac{\partial e_\phi}{\partial \phi}}\dot\phi = -\dot\theta\cos\phi\, \underset{\sim}{e}_R + \dot\theta\sin\phi\, \underset{\sim}{e}_\phi$$

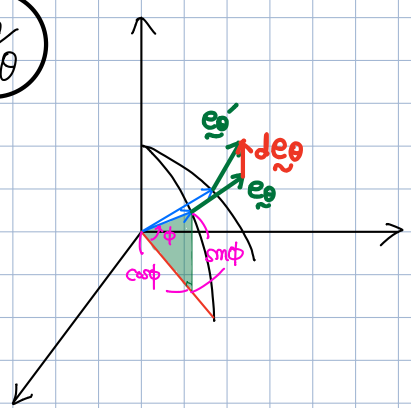
$\dot{\underset{\sim}{e}}_\theta$

i. $\partial e_\theta / \partial R$

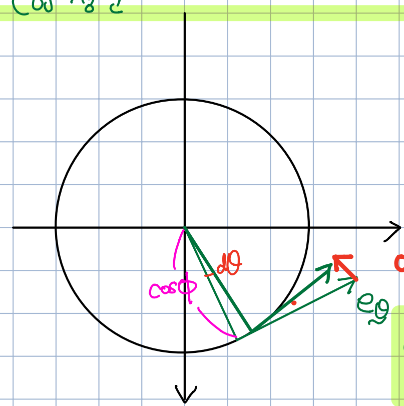


$$\boxed{\frac{\partial e_\theta}{\partial R} = \underset{\sim}{0}}$$

ii. $\partial e_\theta / \partial \theta$



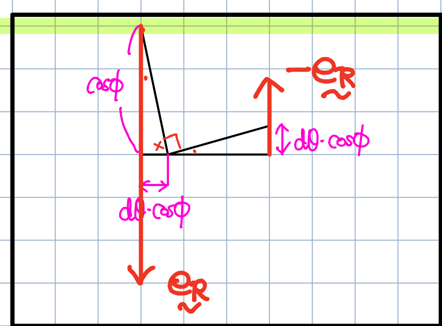
cos 성분



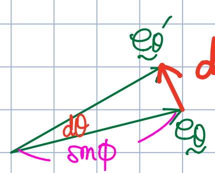
$d\underset{\sim}{e}_\theta = \cos\phi\cdot d\theta\cdot(-\underset{\sim}{e}_R)$

$$\boxed{\frac{de_\theta}{d\theta} = \cos\phi\cdot(-\underset{\sim}{e}_R)}$$

+



sin 성분

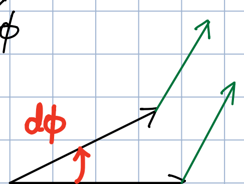


$d\underset{\sim}{e}_\theta = d\theta\cdot\sin\phi\cdot\underset{\sim}{e}_\phi$

$$\boxed{\frac{de_\theta}{d\theta} = \sin\phi\cdot \underset{\sim}{e}_\phi}$$

$$\boxed{\frac{de_\theta}{d\theta} = -\cos\phi\cdot \underset{\sim}{e}_R + \sin\phi\cdot\underset{\sim}{e}_\phi}$$

iii. $\partial e_\theta / \partial \phi$



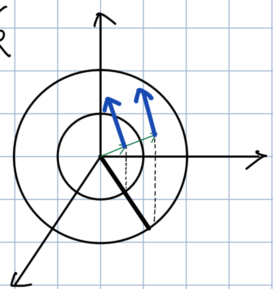
$\frac{\partial e_\theta}{\partial \phi} = \underset{\sim}{0}$

$$\dot{e}_\phi = \frac{d e_\phi(R,\theta,\phi)}{dt} = \frac{1}{dt}\cdot\left\{\frac{\partial e_\phi}{\partial R}dR + \frac{\partial e_\phi}{\partial\theta}\cdot d\theta + \frac{\partial e_\phi}{\partial\phi}d\phi\right\}$$

$$= \underbrace{\cancel{\frac{\partial e_\phi}{\partial R}}}_{=0}\cdot\dot{R} + \underbrace{\frac{\partial e_\phi}{\partial \theta}}_{=-\sin\phi\cdot e_\theta}\cdot\dot{\theta} + \underbrace{\frac{\partial e_\phi}{\partial \phi}}_{=-e_R}\cdot\dot{\phi} = -\dot{\theta}\sin\phi\, e_\theta - \dot{\phi} e_R$$

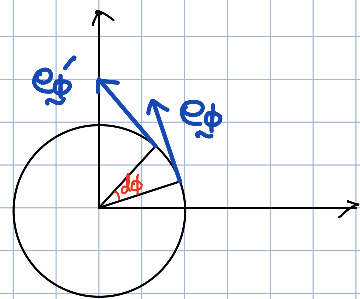
$\dot{e}_\phi$

i. $\partial e_\phi / \partial R$



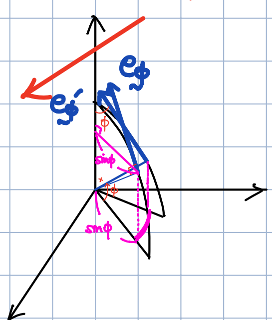
$\boxed{\frac{\partial e_\phi}{\partial R} = 0}$

iii. $\partial e_\phi / \partial \phi$

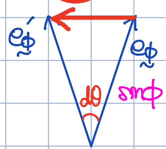


ii. $\partial e_\phi / \partial \theta$

$de_\phi : (-e_\theta$ 방향$)$

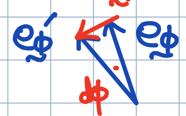


$de_\phi = \sin\phi\cdot d\theta\cdot(-e_\theta)$



$\boxed{de_\phi / \partial\theta = -\sin\phi\cdot e_\theta}$

$de_\phi = d\phi\cdot(-e_R)$



$\boxed{de_\phi / d\phi = -e_R}$

$$\boxed{v = \dot{R}e_R + R\dot{\theta}\cos\phi\, e_\theta + R\dot{\phi}e_\phi}$$

$$a = \dot{v} = \ddot{R}e_R + \dot{R}\dot{e}_R + \qquad = \dot{\theta}\cos\phi\, e_\theta + \dot{\phi}e_\phi \qquad = -\dot{\theta}\cos\phi\, e_R + \dot{\theta}\sin\phi\, e_\phi$$

$$\dot{R}\dot{\theta}\cos\phi\, e_\theta + R\ddot{\theta}\cos\phi\, e_\theta + R\dot{\theta}(-\sin\phi)\cdot\dot{\phi}\cdot e_\theta + R\dot{\theta}\cos\phi\, \dot{e}_\theta +$$

$$\dot{R}\dot{\phi}e_\phi + R\ddot{\phi}e_\phi + R\dot{\phi}\dot{e}_\phi$$

$$= -\dot{\theta}\sin\phi\, e_\theta - \dot{\phi}e_R$$

$$\boxed{\begin{aligned} &= (\ddot{R} - R\dot{\theta}^2\cos^2\phi - R\dot{\phi}^2)e_R + \\ &(2\dot{R}\dot{\theta}\cos\phi + R\ddot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi)e_\theta + \\ &(2\dot{R}\dot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi + R\ddot{\phi})e_\phi \end{aligned}}$$

$= (\ddot{R} - R\dot{\theta}^2\cos^2\phi - R\dot{\phi}^2)e_R +$
$(2\dot{R}\dot{\theta}\cos\phi + R\ddot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi)e_\theta +$
$(2\dot{R}\dot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi + R\ddot{\phi})e_\phi$

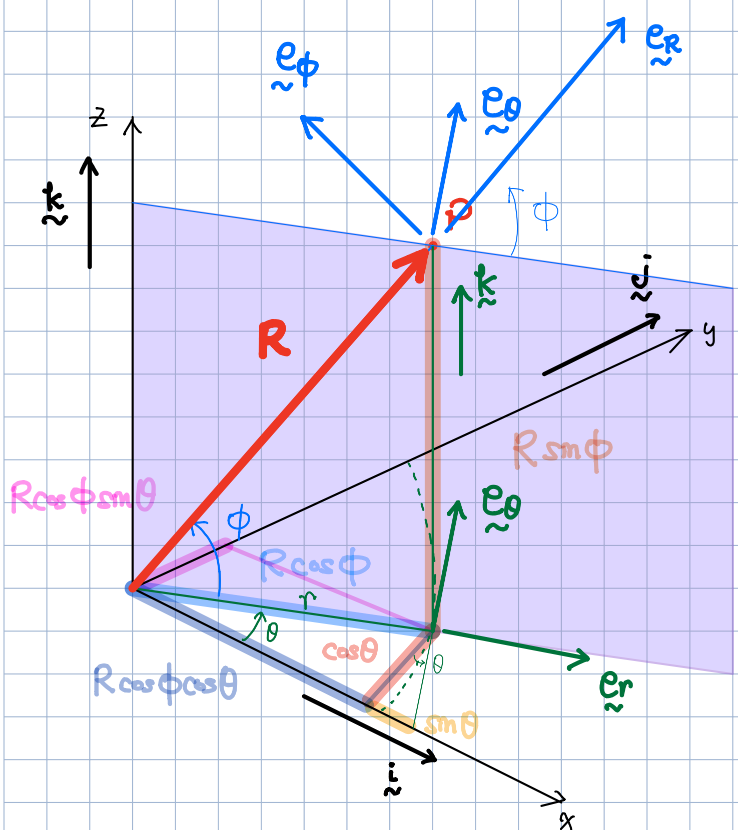
✓ $a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$

✓ $a_\theta = \frac{\cos\phi}{R}\cdot\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi \longleftarrow \frac{d}{dt}(R^2\dot{\theta}) = 2R\dot{R}\dot{\theta} + R^2\ddot{\theta}$

$\frac{1}{R}\frac{d}{dt}(R^2\dot{\theta}) = 2\dot{R}\dot{\theta} + R\ddot{\theta}$

$$= \frac{\cos\phi}{R}(2R\dot{\theta}\dot{R} + R^2\ddot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi$$

3. Spherical Coordinates (R-θ-φ)



$$e_\phi = \cos(\pi/2+\phi)\cdot\cos\theta\, \underset{\sim}{i} + \cos(\pi/2+\phi)\cdot\sin\theta\, \underset{\sim}{j} + \sin(\pi/2+\phi)\, \underset{\sim}{k}$$

$$\underset{\sim}{R} = R\, \underset{\sim}{e_R}$$

$$\left\{\begin{aligned} \underset{\sim}{e_R} &= \cos\phi\cos\theta\, \underset{\sim}{i} + \cos\phi\sin\theta\, \underset{\sim}{j} + \sin\phi\, \underset{\sim}{k} \\ \underset{\sim}{e_\phi} &= -\sin\phi\cos\theta\, \underset{\sim}{i} - \sin\phi\sin\theta\, \underset{\sim}{j} + \cos\phi\, \underset{\sim}{k} \\ \underset{\sim}{e_\theta} &= \sin\theta\,(-\underset{\sim}{i}) + \cos\theta\, \underset{\sim}{j} \end{aligned}\right.$$

$$\dot{\underset{\sim}{R}} = \dot{R}\, \underset{\sim}{e_R} + R\, \dot{\underset{\sim}{e_R}}$$ Velocity

$$\dot{\underset{\sim}{e_R}} = \{(-\sin\phi)\dot\phi\cos\theta + \cos\phi\,(-\sin\theta)\,\dot\theta\}\, \underset{\sim}{i} + \{(-\sin\phi)\dot\phi\sin\theta + \cos\phi\cdot\cos\theta\cdot\dot\theta\}\, \underset{\sim}{j} + \{\cos\phi\,\dot\phi\}\, \underset{\sim}{k}$$

$$= \dot\phi\cdot\underbrace{\{(-\sin\phi)\cos\theta\, \underset{\sim}{i} + (-\sin\phi)\cdot\sin\theta\, \underset{\sim}{j} + \cos\phi\, \underset{\sim}{k}\}}_{=e_\phi} + \dot\theta\cos\phi\cdot\underbrace{\{-\sin\theta\, \underset{\sim}{i} + \cos\theta\, \underset{\sim}{j}\}}_{=e_\theta}$$

$$= \dot\phi\, \underset{\sim}{e_\phi} + \dot\theta\cos\phi\, \underset{\sim}{e_\theta}$$

$$\dot{\underset{\sim}{R}} = \dot{R}\, \underset{\sim}{e_R} + R\, \dot{\underset{\sim}{e_R}}$$

$$\therefore\ \dot{\underset{\sim}{R}} = \dot R\, \underset{\sim}{e_R} + R\,(\dot\phi\, \underset{\sim}{e_\phi} + \dot\theta\cos\phi\, \underset{\sim}{e_\theta})$$

$$= \dot R\, \underset{\sim}{e_R} + R\dot\theta\cos\phi\, \underset{\sim}{e_\theta} + R\dot\phi\, \underset{\sim}{e_\phi}$$ Ans.

$$\underset{\sim}{v} = \dot R\, \underset{\sim}{e_R} + R\,(\cos\phi\,\dot\theta\, \underset{\sim}{e_\theta} + \dot\phi\, \underset{\sim}{e_\phi})$$

$$= \dot R\, \underset{\sim}{e_R} + R\dot\theta\cos\phi\, \underset{\sim}{e_\theta} + R\dot\phi\, \underset{\sim}{e_\phi}$$

$$\begin{cases} e_R = \cos\phi\cos\theta\, i + \cos\phi\sin\theta\, j + \sin\phi\, k \\ e_\phi = -\sin\phi\cos\theta\, i - \sin\phi\sin\theta\, j + \cos\phi\, k \\ e_\theta = \sin\theta\,(-i) + \cos\theta\, j \end{cases}$$

$$\boxed{\dot{R} = \dot{R}\, e_R + R\,\dot{e}_R}$$

$$\ddot{R} = \ddot{R}\, e_R + \dot{R}\,\dot{e}_R + \dot{R}\,\dot{e}_R + R\,\ddot{e}_R$$

$$\boxed{= \ddot{R}\, e_R + 2\dot{R}\,\dot{e}_R + R\,\ddot{e}_R}$$: acceleration

$$\dot{e}_R = \dot\phi\, e_\phi + \dot\theta\cos\phi\, e_\theta \longrightarrow \ddot{e}_R = \ddot\phi\, e_\phi + \dot\phi\,\dot{e}_\phi + \ddot\theta\cos\phi\, e_\theta + \dot\theta(-\sin\phi)\dot\phi\, e_\theta + \dot\theta(\cos\phi)\,\dot{e}_\theta$$

$$= \ddot\phi\, e_\phi + \dot\phi(-\dot\phi\, e_R - \sin\phi\,\dot\theta\, e_\theta) + \ddot\theta\cos\phi\, e_\theta - \dot\theta\cdot\dot\phi\sin\phi\, e_\theta + \dot\theta\cos\phi(-\cos\theta\,\dot\theta\, i - \sin\theta\,\dot\theta\, j)$$

$$\dot{e}_\theta = -\cos\theta\cdot\dot\theta\, i - \sin\theta\cdot\dot\theta\, j$$

$$\begin{aligned} \dot{e}_\phi = &-\cos\phi\cdot\dot\phi\cos\theta\, i - \sin\phi\cdot(-\sin\theta)\,\dot\theta\, i \\ &-\cos\phi\,\dot\phi\sin\theta\, j - \sin\phi\cdot(\cos\theta)\,\dot\theta\, j \\ &-\sin\phi\,\dot\phi\, k \\ = &-\dot\phi\{\cos\phi\cos\theta\, i + \cos\phi\sin\theta\, j + \sin\phi\, k\} \\ &-\sin\phi\cdot\dot\theta\,\{-\sin\theta\, i + \cos\theta\, j\} \\ = &-\dot\phi\, e_R - \sin\phi\,\dot\theta\, e_\theta \end{aligned}$$

$$-R\dot\theta^2\cos^2\phi\left(\frac{\cos\theta}{\cos\phi}\, i\right) - R\dot\theta^2\cos\phi\sin\phi\left(\frac{\sin\theta}{\sin\phi}\, j\right)$$

$= e_R$ $\qquad$ $= -e_\phi$

$\because\ i\cos\theta = e_R\cos\phi$

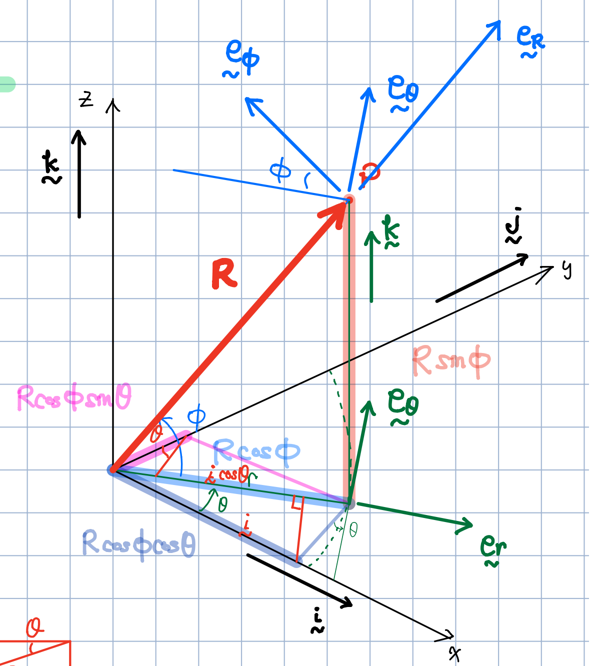
$\because\ j\sin\theta = -\sin\phi\, e_\phi$
$= e_R\cos\phi$
$= e_\phi\cos(\pi/2+\phi)$
$= -\sin\phi\, e_\phi$

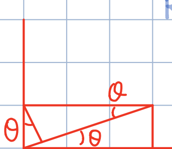
$$\boxed{\ddot{R}\, e_R + 2\dot{R}\,\dot{e}_R + R\,\ddot{e}_R}$$

$$\begin{aligned} \therefore\ \ddot{R} = &\ \ddot{R}\, e_R + 2\dot{R}(\dot\phi\, e_\phi + \dot\theta\cos\phi\, e_\theta) \\ &+ R\{\ddot\phi\, e_\phi + \dot\phi(-\dot\phi\, e_R - \sin\phi\,\dot\theta\, e_\theta) + \ddot\theta\cos\phi\, e_\theta \\ &\qquad - \dot\theta\cdot\dot\phi\sin\phi\, e_\theta + \dot\theta\cos\phi(-\cos\theta\,\dot\theta\, i - \sin\theta\,\dot\theta\, j)\} \\ = &\ \ddot{R}\, e_R + 2\dot{R}\dot\phi\, e_\phi + 2\dot{R}\dot\theta\cos\phi\, e_\theta \\ &+ R\ddot\phi\, e_\phi - R\dot\phi^2\, e_R - R\dot\theta\dot\phi\sin\phi\, e_\theta + R\ddot\theta\cos\phi\, e_\theta \\ &- R\dot\theta\dot\phi\sin\phi\, e_\theta - R\dot\theta^2\cos\phi\cos\theta\, i - R\dot\theta^2\cos\phi\sin\theta\, j \\ = &\ (\ddot{R} - R\dot\theta\cos^2\phi - R\dot\phi^2)\, e_R \\ &+ (2\dot{R}\dot\theta\cos\phi + R\ddot\theta\cos\phi - 2R\dot\theta\dot\phi\sin\phi)\, e_\theta \\ &+ (2\dot{R}\dot\phi + R\dot\theta\cos\phi\sin\phi + R\ddot\phi)\, e_\phi \end{aligned}$$

Ans.

$\because\ i\cos\theta = e_R\cos\phi$





$$a = (\ddot{R} - R\dot\theta^2\cos^2\phi - R\dot\phi^2)\, e_R + (2\dot{R}\dot\theta\cos\phi + R\ddot\theta\cos\phi - 2R\dot\theta\dot\phi\sin\phi)\, e_\theta + (2\dot{R}\dot\phi + R\dot\theta^2\sin\phi\cos\phi + R\ddot\phi)\, e_\phi$$