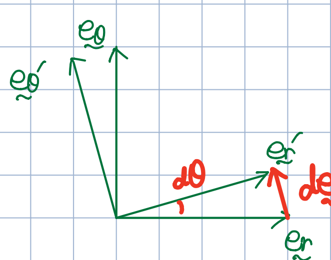


$de_r = d\theta \cdot (e_\theta)$

$$\frac{de_r}{d\theta} = e_\theta$$

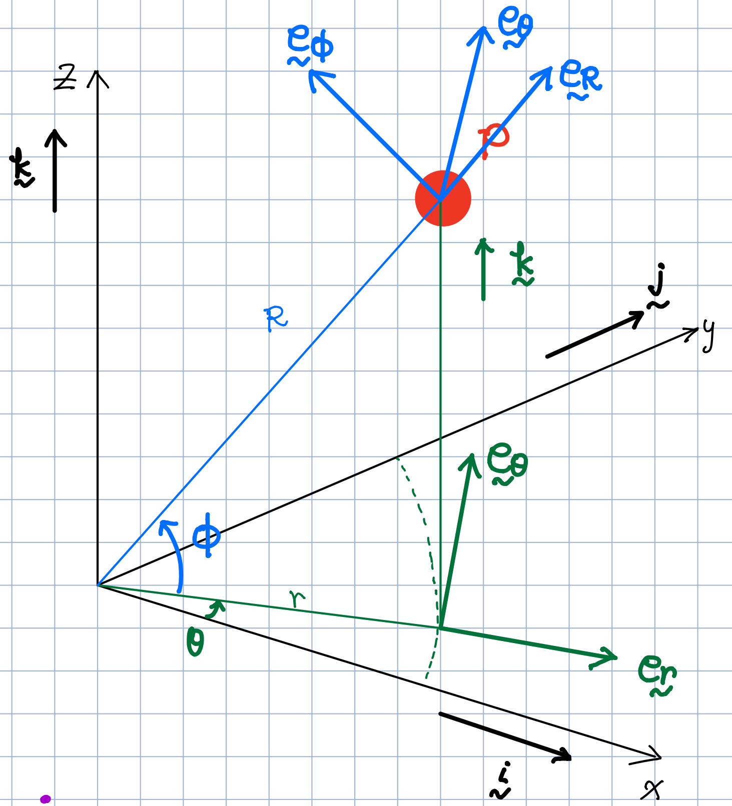
$\dot{e}_r = \dot{\theta} \cdot e_\theta$

$$\dot{e}_r = \frac{d e_r(r, \theta, k)}{dt} = \frac{1}{dt} \cdot \left\{ \underbrace{\cancel{\frac{\partial e_r}{\partial r}}}_{=0} \cdot dr + \underbrace{\frac{\partial e_r}{\partial \theta}}_{= e_\theta} d\theta + \underbrace{\cancel{\frac{\partial e_r}{\partial k}}}_{=0} \cdot dk \right\}$$

$$= \frac{d\theta}{dt} \cdot e_\theta$$

$$= \dot{\theta}\, e_\theta$$

$$\boxed{\begin{aligned} &\frac{de_r}{d\theta} = e_\theta \\ &\frac{de_r}{dt} \cdot \frac{dt}{d\theta} = e_\theta \\ &\dot{e}_r = \frac{d\theta}{dt} \cdot e_\theta = \dot{\theta} e_\theta \end{aligned}}$$



3. Spherical Coordinate (R-θ-φ)

$$\boxed{\underset{\sim}{R} = R\, e_R}$$

$$v = \dot{\underset{\sim}{R}} = \dot R\, e_R + R\, \dot e_R$$

$$\dot e_R = \frac{d e_R(R,\theta,\phi)}{dt} = \cos\phi \cdot \dot\theta\, e_\theta + \dot\phi\, e_\phi$$

$$= \frac{1}{dt}\left\{\frac{\partial e_R}{\partial R} dR + \frac{\partial e_R}{\partial \theta}\cdot d\theta + \frac{\partial e_R}{\partial \phi}\cdot d\phi\right\}$$

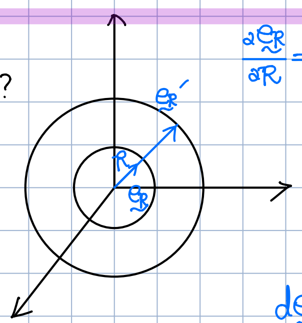
$$= \underbrace{\cancel{\frac{\partial e_R}{\partial R}}}_{=0}\cdot \dot R + \underbrace{\frac{\partial e_R}{\partial \theta}}_{=\cos\phi \cdot e_\theta}\cdot \dot\theta + \underbrace{\frac{\partial e_R}{\partial \phi}}_{=e_\phi}\cdot \dot\phi$$

$$v = \dot R\, e_R + R(\cos\phi \cdot \dot\theta\, e_\theta + \dot\phi\, e_\phi)$$

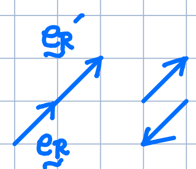
$$\boxed{= \dot R\, e_R + R\dot\theta\cos\phi\, e_\theta + R\dot\phi\, e_\phi}$$

$\dot e_R$

i. $\partial e_R / \partial R$?

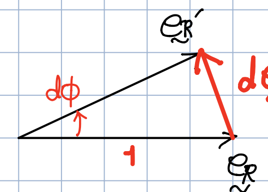


$\frac{\partial e_R}{\partial R} = 0$



$$d e_R = e_R' - e_R$$
$$= \nearrow\swarrow$$
$$= 0$$

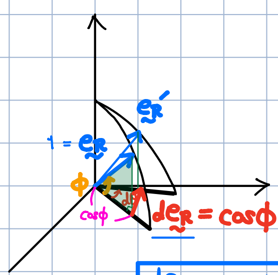
iii. $\partial e_R / \partial \phi$



$$d e_R = d\phi \cdot e_\phi$$

$$\boxed{d e_R / d\phi = e_\phi}$$

ii. $\partial e_R / \partial \theta$



$$d e_R = \cos\phi \cdot d\theta \cdot e_\theta$$

$$\boxed{d e_R / d\theta = \cos\phi \cdot e_\theta}$$

$$\boxed{v = \dot R\, e_R + R\dot\theta\cos\phi\, e_\theta + R\dot\phi\, e_\phi}$$

$$a = \dot v = \ddot R\, e_R + \dot R\, \underbrace{\dot e_R}_{= \dot\theta\cos\phi\, e_\theta + \dot\phi\, e_\phi} +$$

$$\dot R\dot\theta\cos\phi\, e_\theta + R\ddot\theta\cos\phi\, e_\theta + R\dot\theta(-\sin\phi)\cdot\dot\phi\cdot e_\theta + R\dot\theta\cos\phi\, \dot e_\theta +$$

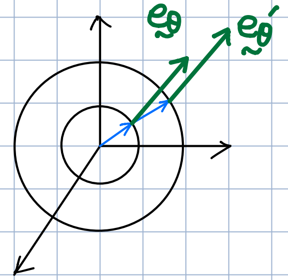
$$\dot R\dot\phi\, e_\phi + R\ddot\phi\, e_\phi + R\dot\phi\, \dot e_\phi$$

$$\dot{e}_\theta = \frac{d e_\theta(R,\theta,\phi)}{dt} = \frac{1}{dt}\cdot\left\{\frac{\partial e_\theta}{\partial R}\cdot dR + \frac{\partial e_\theta}{\partial \theta}\cdot d\theta + \frac{\partial e_\phi}{\partial \phi}d\phi\right\}$$

$$= \underbrace{\cancel{\frac{\partial e_\theta}{\partial R}}}_{=0}\cdot \dot{R} + \underbrace{\frac{\partial e_\theta}{\partial \theta}}_{=-\cos\phi\cdot e_R+\sin\phi\, e_\phi}\dot{\theta} + \cancel{\frac{\partial e_\phi}{\partial \phi}}\dot{\phi} = -\dot{\theta}\cos\phi\, e_R + \dot{\theta}\sin\phi\, e_\phi$$

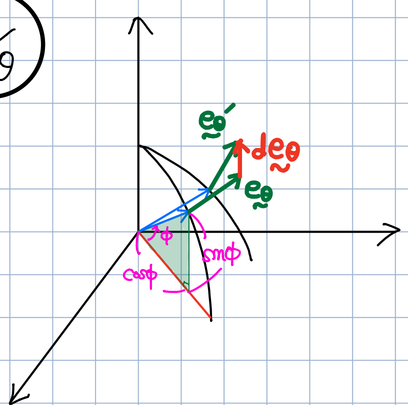
$\dot{e}_\theta$

i. $\partial e_\theta / \partial R$

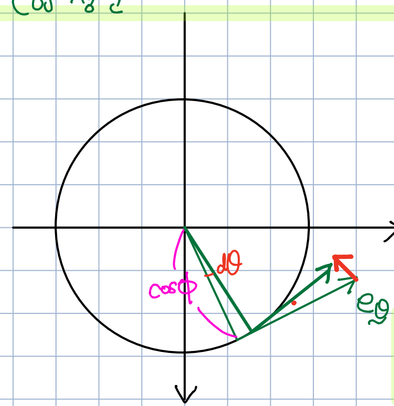


$$\boxed{\partial e_\theta / \partial R = 0}$$

ii. $\partial e_\theta / \partial \theta$



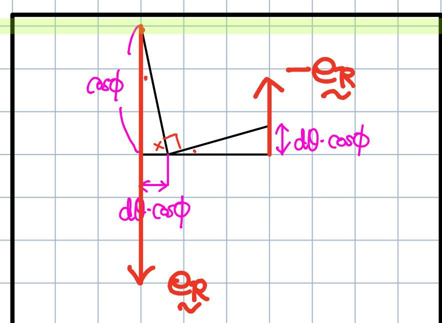
cos 성분



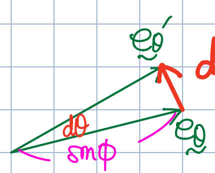
$de_\theta = \cos\phi \cdot d\theta \cdot (-e_R)$

$$\boxed{de_\theta / d\theta = \cos\phi\cdot(-e_R)}$$

\+



sin 성분

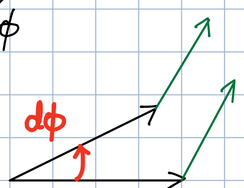


$de_\theta = d\theta\cdot\sin\phi\cdot e_\phi$

$$\boxed{de_\theta / d\theta = \sin\phi\cdot e_\phi}$$

$$\boxed{de_\theta / d\theta = -\cos\phi\cdot e_R + \sin\phi\cdot e_\phi}$$

iii. $\partial e_\theta / \partial \phi$



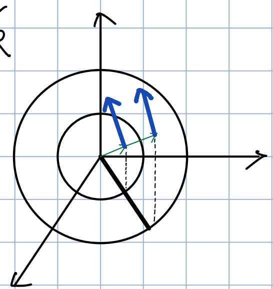
$\partial e_\theta / \partial \phi = 0$

$$\dot{e}_\phi = \frac{d e_\phi(R,\theta,\phi)}{dt} = \frac{1}{dt}\cdot\left\{\frac{\partial e_\phi}{\partial R}dR + \frac{\partial e_\phi}{\partial\theta}\cdot d\theta + \frac{\partial e_\phi}{\partial\phi}d\phi\right\}$$

$$= \underbrace{\cancel{\frac{\partial e_\phi}{\partial R}}}_{=0}\cdot\dot R + \underbrace{\frac{\partial e_\phi}{\partial \theta}}_{=-\sin\phi\cdot e_\theta}\cdot\dot\theta + \underbrace{\frac{\partial e_\phi}{\partial\phi}}_{=-e_R}\cdot\dot\phi = -\dot\theta\sin\phi\, e_\theta - \dot\phi\, e_R$$

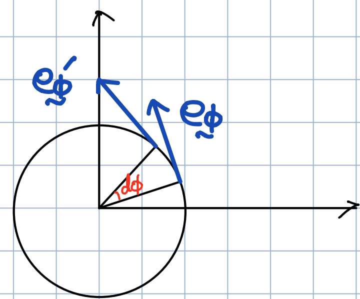
$\dot{e}_\phi$

i. $\partial e_\phi / \partial R$



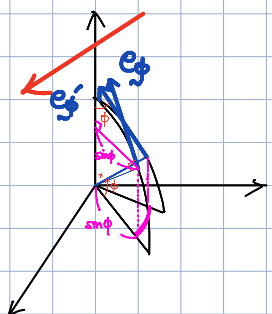
$$\boxed{\frac{\partial e_\phi}{\partial R} = 0}$$

iii. $\partial e_\phi / \partial \phi$

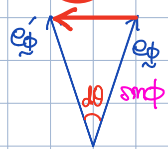


ii. $\partial e_\phi / \partial \theta$

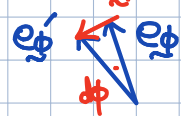
$de_\phi : (-e_\theta$ 방향$)$



$de_\phi = \sin\phi\cdot d\theta\cdot(-e_\theta)$



$de_\phi = d\phi\cdot(-e_R)$



$$\boxed{de_\phi / d\theta = -\sin\phi\cdot e_\theta}$$

$$\boxed{de_\phi / d\phi = -e_R}$$

$$\boxed{v = \dot R e_R + R\dot\theta\cos\phi\, e_\theta + R\dot\phi\, e_\phi}$$

$$a = \dot v = \ddot R e_R + \dot R\,\dot e_R + \quad = \dot\theta\cos\phi\, e_\theta + \dot\phi\, e_\phi$$

$$\dot R\dot\theta\cos\phi\, e_\theta + R\ddot\theta\cos\phi\, e_\theta + R\dot\theta(-\sin\phi)\cdot\dot\phi\cdot e_\theta + R\dot\theta\cos\phi\,\dot e_\theta +$$

$$= -\dot\theta\cos\phi\, e_R + \dot\theta\sin\phi\, e_\phi$$

$$\dot R\dot\phi\, e_\phi + R\ddot\phi\, e_\phi + R\dot\phi\,\dot e_\phi$$

$$= -\dot\theta\sin\phi\, e_\theta - \dot\phi\, e_R$$

$$\boxed{\begin{aligned} &= (\ddot R - R\dot\theta^2 - R\dot\phi^2)e_R + \\ &(2\dot R\dot\theta\cos\phi + R\ddot\theta\cos\phi - 2R\dot\theta\dot\phi\sin\phi)e_\theta + \\ &(2\dot R\dot\phi + R\dot\theta^2\sin\phi\cos\phi + R\ddot\phi)e_\phi \end{aligned}}$$

✓ $a_R = \ddot R - R\dot\phi^2 - R\dot\theta^2$

✓ $a_\theta = \frac{\cos\phi}{R}\cdot\frac{d}{dt}(R^2\dot\theta) - 2R\dot\theta\dot\phi\sin\phi \longleftarrow \frac{d}{dt}(R^2\dot\theta) = 2R\dot R\dot\theta + R^2\ddot\theta$

$= \frac{\cos\phi}{R}(2R\dot\theta\dot R + R^2\ddot\theta) - 2R\dot\theta\dot\phi\sin\phi \qquad \frac{1}{R}\frac{d}{dt}(R^2\dot\theta) = 2\dot R\dot\theta + R\ddot\theta$

✓ $a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi$ ← $\frac{d}{dt}(R^2\dot{\phi}) = 2R\dot{R}\dot{\phi} + R^2\ddot{\phi}$

$= \frac{1}{R}(2R\dot{\phi}\dot{R} + R^2\ddot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi$ $\quad \frac{1}{R}\cdot\frac{d}{dt}(R^2\dot{\phi}) = 2\dot{R}\dot{\phi} + R\ddot{\phi}$

$= 2\dot{\phi}\dot{R} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$