

$$\underline{a} = \frac{d\underline{v}}{dt} \leftarrow \underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$$

$$= \ddot{r}\underline{e}_r + \dot{r}\dot{\underline{e}}_r + \dot{r}\dot{\theta}\underline{e}_\theta + r\ddot{\theta}\underline{e}_\theta + r\dot{\theta}\dot{\underline{e}}_\theta \leftarrow \dot{\underline{e}}_r = \dot{\theta}\underline{e}_\theta, \quad \dot{\underline{e}}_\theta = -\dot{\theta}\underline{e}_r$$

$$= \ddot{r}\underline{e}_r + \dot{r}\dot{\theta}\underline{e}_\theta + \dot{r}\dot{\theta}\underline{e}_\theta + r\ddot{\theta}\underline{e}_\theta - r\dot{\theta}^2\underline{e}_r$$

$$= (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$\bullet \underline{a}_r = \ddot{r} - r\dot{\theta}^2$$

$\underline{a}_r$: scalar value

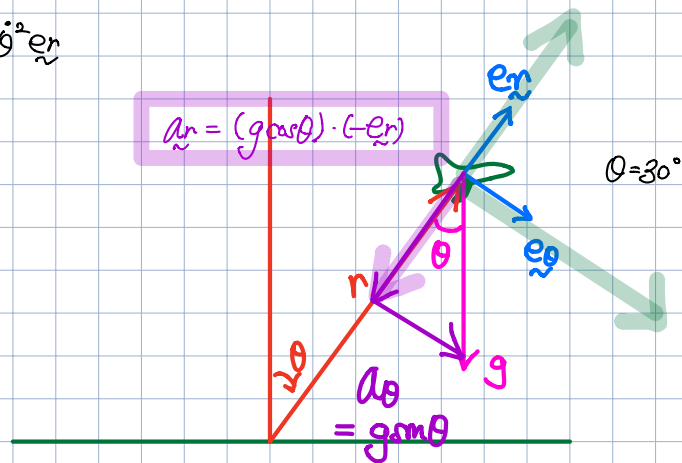
$$\underline{a}_r = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r$$

$$\underline{a}_r = (g\cos\theta) \cdot (-\underline{e}_r) = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r$$

$$\ddot{r} = -g\cos\theta + r\dot{\theta}^2$$

$$= (-31.4 \text{ ft/s}^2) \cdot \cos 30^\circ + (25 \times 10^4 \text{ ft}) \cdot (0.8 \text{ deg/sec} \cdot \frac{\pi \text{ rad}}{180 \text{ deg}})^2$$

$$= \underline{21.5 \text{ ft/s}^2} \text{ Ans.}$$



Dynamics of Particles

1. Introduction to Dynamics : Newton's Laws , units , dimensions
2. Kinematics of Particles
3. Kinetics of Particles
4. Kinetics of Systems of Particles

Dynamics of Rigid Bodies

1. Plane Kinematics of Rigid Bodies
2. Plane Kinetics of Rigid Bodies
3. 3D Dynamics of Rigid Bodies
4. Vibration

Chapter 2. Kinematics of Particles

Rectilinear Motion

Plane Curvilinear Motion

Rectangular coordinate ($x-y$)

Normal and Tangential Coordinate ($n-t$)

Polar Coordinates ($r-\theta$)

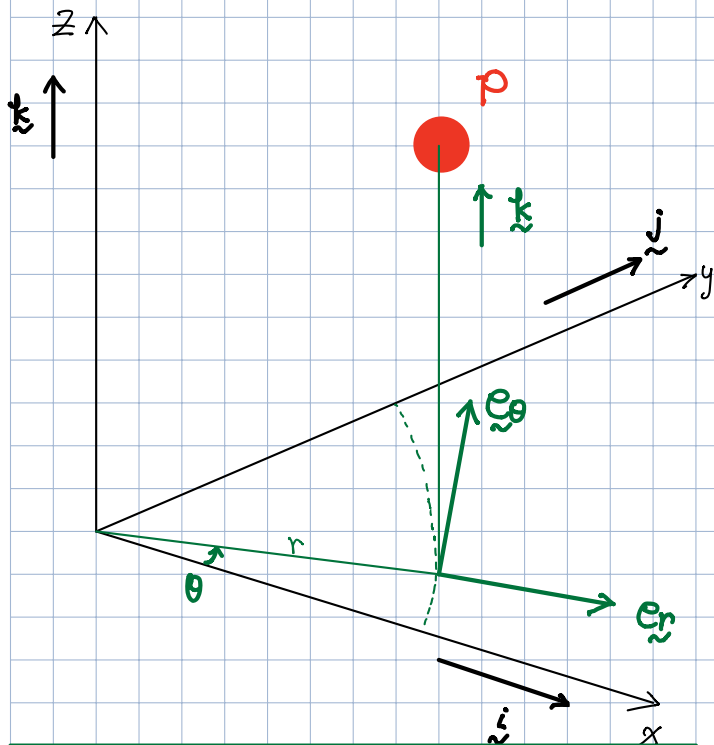
Space Curvilinear Motion

Rectangular Coordinates ($x-y-z$)

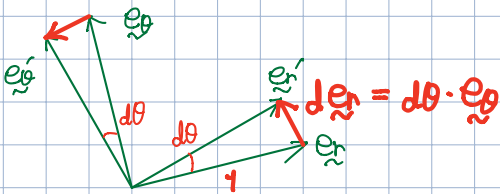
Cylindrical Coordinates ($r-\theta-z$)

Spherical Coordinates ($R-\theta-\phi$)

Constrained Motion of Connected Particles



$$de_\theta = d\theta \cdot (-e_r)$$



$$de_r = d\theta \cdot e_\theta$$

$$de_\theta = -d\theta \cdot e_r$$

$$\frac{de_r}{dt} = \frac{d\theta}{dt} \cdot e_\theta$$

$$\frac{de_\theta}{dt} = -\frac{d\theta}{dt} \cdot e_r$$

$$\dot{e}_r = \dot{\theta} \cdot e_\theta$$

$$\dot{e}_\theta = -\dot{\theta} \cdot e_r$$

1. Rectangular Coordinates (x-y-z)

$$\underline{R} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$\begin{aligned}\underline{v} = \dot{\underline{R}} &= \frac{d}{dt}(x\underline{i} + y\underline{j} + z\underline{k}) \\ &= (\dot{x}\underline{i} + \cancel{x\dot{\underline{i}}}) + (\dot{y}\underline{j} + \cancel{y\dot{\underline{j}}}) + (\dot{z}\underline{k} + \cancel{z\dot{\underline{k}}}) \\ &= \dot{x}\underline{i} + \dot{y}\underline{j} + \dot{z}\underline{k}\end{aligned}$$

$$\underline{a} = \dot{\underline{v}} = \ddot{x}\underline{i} + \ddot{y}\underline{j} + \ddot{z}\underline{k}$$

2. Cylindrical Coordinate (r-theta-z)

$$\underline{R} = r\underline{e}_r + z\underline{k}$$

$$\begin{aligned}\underline{v} = \dot{\underline{R}} &= \frac{d}{dt}(r\underline{e}_r + z\underline{k}) \\ &= (\dot{r}\underline{e}_r + r\dot{\underline{e}}_r) + (\dot{z}\underline{k} + \cancel{z\dot{\underline{k}}}) \\ &= \dot{r}\underline{e}_r + r\dot{\underline{e}}_r + \dot{z}\underline{k} \\ &= \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}\end{aligned}$$

$$v_r = \dot{r}$$

$$v_\theta = r\dot{\theta}$$

$$v_z = \dot{z}$$

$$\rightarrow v = \sqrt{v_r^2 + v_\theta^2 + v_z^2}$$

$$\begin{aligned}\underline{a} = \dot{\underline{v}} &= \ddot{r}\underline{e}_r + r\dot{\underline{e}}_r + \dot{r}\dot{\underline{e}}_r + r\ddot{\theta}\underline{e}_\theta + r\dot{\theta}\dot{\underline{e}}_\theta + r\dot{\theta}\dot{\underline{e}}_\theta + r\dot{\theta}\dot{\underline{e}}_\theta + \ddot{z}\underline{k} + \cancel{\dot{z}\dot{\underline{k}}} \\ &= (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (2r\dot{\theta} + r\ddot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}\end{aligned}$$