Example > 2.5

The curvilinear motion : $v_x = 50-16t$ [m/s]
$y = 100-4t^2$ [m]

$x=0$ when $t=0$

Determine its velocity and accleration when $y=0$.

y=0 가 되는 시간?

$y = 100-4t^2 = 0$
$4t^2 = 100$
$t^2 = 25$
$t = 5$ sec

s
$v = ds/dt$
$a = dv/dt$

$\rightarrow$

x	y
$v_x = dx/dt$	$v_y = dy/dt$
$a_x = dv_x/dt$	$a_y = dv_y/dt$

$\underset{\sim}{v}, \underset{\sim}{a} = ?$

$\underset{\sim}{v} = v_x \underset{\sim}{i} + v_y \underset{\sim}{j}$

$v_x = 50-16t$
$v_y = ?$
$v = ds/dt$
$v_y = dy/dt \leftarrow y = 100-4t^2$
$= -8t$

$\therefore \underset{\sim}{v} = (50-16t)\underset{\sim}{i} + (-8t)\underset{\sim}{j}$

$t=5$ sec $\longrightarrow$ $v_{t=5} = (50-16\times5)\underset{\sim}{i} + (-8\times5)\underset{\sim}{j}$

$= -30i - 40j$ Ans.

$\underset{\sim}{a} = a_x \underset{\sim}{i} + a_y \underset{\sim}{j}$

$a_x = dv_x/dt \quad \leftarrow \quad v_x = 50 - 16t$

$= -16$

$a_y = dv_y/dt \quad \leftarrow \quad v_y = -8t$

$= -8$

$\therefore \underset{\sim}{a} = (-16)\underset{\sim}{i} + (-8)\underset{\sim}{j}$ Ans.

Dynamics of Particle

1. Introduction to Dynamics : Newton's Laws, units, dimensions.
2. Kinematics of Particles
3. Kinetics of Particles
4. Kinetics of Systems of Particles

Dynamics of Rigid Bodies

1. Plane Kinematics of Rigid Bodies
2. Plane Kinetics of Rigid Bodies
3. 3D Dynamics of Rigid Bodies
4. Vibrations

Chapter 2. Kinematics of Particle

<u>Rectilinear Motion</u>

<u>Plane Curvilinear Motion</u>

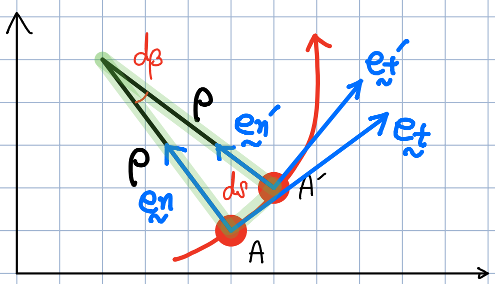
- Rectangular coordinate $(x-y)$
- Normal - Tangent Coordinate $(n-t)$
- Polar coordinate $(r-\theta)$

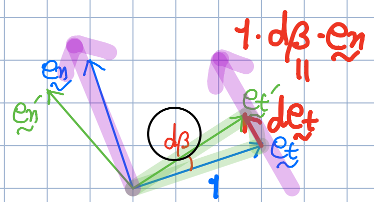
<u>Space Curvilinear Motion</u>

- Rectangular coordinate $(x-y-z)$
- Cylindrical coordinate $(r-\theta-z)$
- Spherical coordinate $(R-\theta-\phi)$

<u>Constrained Motion of Connected Particles</u>

Normal and Tangential Coordinates (n-t)





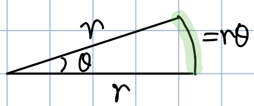
$$d\underset{\sim}{e_t} = \underset{\sim}{e_n} \cdot d\beta \quad \longleftarrow \quad dt\text{로 나누면.}$$

$$\frac{d\underset{\sim}{e_t}}{dt} = \underset{\sim}{e_n} \cdot \frac{d\beta}{dt}$$

$$\dot{\underset{\sim}{e_t}} = \dot{\beta}\,\underset{\sim}{e_n}$$

1) $\underset{\sim}{v}$

$$\underset{\sim}{v} = v\,\underset{\sim}{e_t}$$



$$v = \frac{ds}{dt} \quad \longleftarrow \quad ds = \rho \cdot d\beta$$

$$= \rho \frac{d\beta}{dt}$$

$$= \rho\dot{\beta}$$

$$\boxed{\underset{\sim}{v} = \rho\dot{\beta}\,\underset{\sim}{e_t}}$$

2) $\underset{\sim}{a}$

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} \quad \longleftarrow \quad \underset{\sim}{v} = v\underset{\sim}{e_t}$$

$$= \frac{d}{dt}(\underset{\sim}{v})$$

$$= \frac{d}{dt}(v\underset{\sim}{e_t})$$

$$= \dot{v}\underset{\sim}{e_t} + v\dot{\underset{\sim}{e_t}} \quad \longleftarrow \quad \dot{\underset{\sim}{e_t}} = \dot{\beta}\underset{\sim}{e_n}$$

$$= \dot{v}\underset{\sim}{e_t} + v\dot{\beta}\underset{\sim}{e_n} \quad \longleftarrow \quad v = \rho\dot{\beta}, \quad \dot{\beta} = v/\rho$$

$$\boxed{\underset{\sim}{a} = \frac{v^2}{\rho}\underset{\sim}{e_n} + \dot{v}\underset{\sim}{e_t}}$$

$$a_n = \frac{v^2}{\rho} \quad \longleftarrow \quad v = \rho\dot{\beta}$$

$$= \frac{\rho^2\dot{\beta}^2}{\rho}$$

$$= \rho\dot{\beta}^2 \quad \longleftarrow \quad v = \rho\dot{\beta}$$

$$= v\dot{\beta}$$

,

$$a_t = \dot{v} = \ddot{s}$$

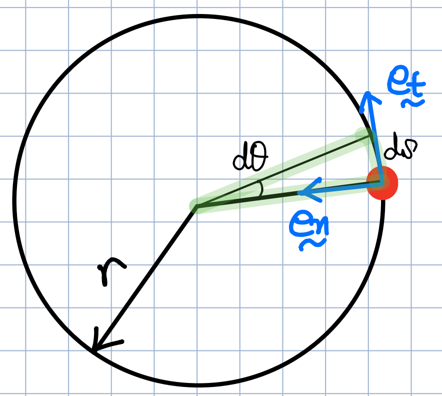
$$= \frac{dv}{dt} \quad \longleftarrow \quad v = \rho\dot{\beta}$$

$$= \frac{d}{dt}(\rho\dot{\beta})$$

$$= \dot{\rho}\dot{\beta} + \rho\ddot{\beta}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

Circular Motion (n-t coordinate)



1) $\underset{\sim}{v} = ds/dt \leftarrow ds = r \cdot d\theta$

$= (r \cdot d\theta)/dt$

$= r \cdot d\theta/dt = r\dot{\theta}$ Ans.

2) $\underset{\sim}{a} = v^2/\rho \, \underset{\sim}{e_n} + \dot{v} \underset{\sim}{e_t}$

$$\boxed{\underset{\sim}{a} = v^2/\rho \, \underset{\sim}{e_n} + \dot{v} \underset{\sim}{e_t}}$$

$a_n = v^2/r \leftarrow v = r\dot{\theta}$

$= r^2\dot{\theta}^2/r = r\dot{\theta}^2$ Ans.

$a_t = \dot{v}$

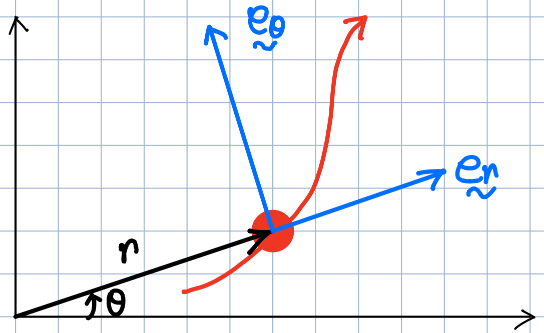
$= d/dt(v) \leftarrow v = r\dot{\theta}$

$= d/dt(r\dot{\theta})$

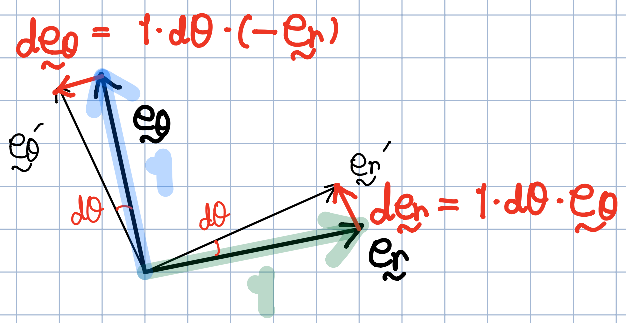
$= \cancel{\dot{r}\dot{\theta}} + r\ddot{\theta} \leftarrow r = const$

$a_t = r\ddot{\theta}$ Ans.

Polar Coordinate $(r-\theta)$



$\mathbf{r} = r\,\mathbf{e}_r$



$$\boxed{\begin{aligned} d\mathbf{e}_r &= \mathbf{e}_\theta\,d\theta \\ d\mathbf{e}_\theta &= -\mathbf{e}_r\,d\theta \end{aligned}}$$

$\longrightarrow \ d\mathbf{e}_r/d\theta = \mathbf{e}_\theta$

$\longrightarrow \ d\mathbf{e}_\theta/d\theta = -\mathbf{e}_r$

$d\mathbf{e}_r = \mathbf{e}_\theta\,d\theta \ \longleftarrow$ dt로 나누면

$d\mathbf{e}_r/dt = \mathbf{e}_\theta\,d\theta/dt$

$\therefore \dot{\mathbf{e}}_r = \mathbf{e}_\theta\cdot\dot\theta$

$d\mathbf{e}_\theta = -\mathbf{e}_r\cdot d\theta$

$\dot{\mathbf{e}}_\theta = -\mathbf{e}_r\,\dot\theta$

1) $\mathbf{v}$

$\mathbf{r} = r\mathbf{e}_r \longrightarrow \mathbf{v} = \dot{\mathbf{r}} = d\mathbf{r}/dt$

$= d/dt\,(r\mathbf{e}_r)$

$= \dot r\mathbf{e}_r + r\dot{\mathbf{e}}_r \ \longleftarrow \ \dot{\mathbf{e}}_r = \mathbf{e}_\theta\cdot\dot\theta$

$$\boxed{\mathbf{v} = \dot r\mathbf{e}_r + r\dot\theta\mathbf{e}_\theta}$$

2) $\underset{\sim}{a}$

$$\underset{\sim}{a} = \dot{\underset{\sim}{v}} \leftarrow \underset{\sim}{v} = \dot{r}\underset{\sim}{e_r} + r\dot{\theta}\underset{\sim}{e_\theta}$$

$$= \frac{dv}{dt}$$

$$= \frac{d}{dt}(\dot{r}\underset{\sim}{e_r} + r\dot{\theta}\underset{\sim}{e_\theta})$$

$$= \ddot{r}\underset{\sim}{e_r} + \dot{r}\dot{\underset{\sim}{e_r}} + \dot{r}\dot{\theta}\underset{\sim}{e_\theta} + r\ddot{\theta}\underset{\sim}{e_\theta} + r\dot{\theta}\dot{\underset{\sim}{e_\theta}} \leftarrow \dot{\underset{\sim}{e_r}} = \underset{\sim}{e_\theta}\cdot\dot{\theta}$$

$$\dot{\underset{\sim}{e_\theta}} = -\underset{\sim}{e_r}\dot{\theta}$$

$$= \ddot{r}\underset{\sim}{e_r} + \dot{r}\dot{\theta}\underset{\sim}{e_\theta} + \dot{r}\dot{\theta}\underset{\sim}{e_\theta} + r\ddot{\theta}\underset{\sim}{e_\theta} - r\dot{\theta}^2 e_r$$

$$= \boxed{(\ddot{r} - r\dot{\theta}^2)\underset{\sim}{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underset{\sim}{e_\theta}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ ⟶ r를 곱하면, $r\cdot a_\theta = r^2\ddot{\theta} + 2\dot{r}\dot{\theta}\cdot r$

$$= r^2\ddot{\theta} + 2r\dot{r}\dot{\theta}$$

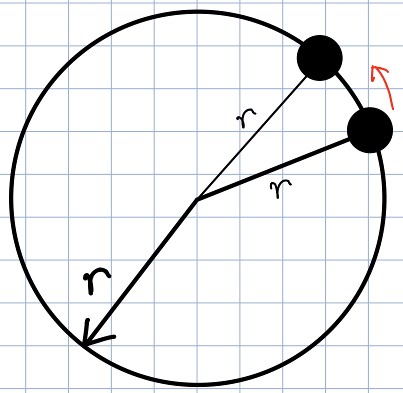
$a = \sqrt{a_r^2 + a_\theta^2}$

$$= \frac{d}{dt}(r^2\dot{\theta})$$

$$2r\dot{r}\dot{\theta} + r^2\dot{\theta}^2$$

$$\boxed{a_\theta = \frac{1}{r}\cdot\frac{d}{dt}(r^2\dot{\theta})}$$

Circular Motion (Polar Coordinate)



$r = \text{const}$

$\dot{r} = 0$

$\ddot{r} = 0$

Polar coordinate

$$\boxed{\underset{\sim}{v} = \cancel{\dot{r}\,\underset{\sim}{e_r}} + r\dot{\theta}\,\underset{\sim}{e_\theta}} \longrightarrow v_r = 0, \quad v_\theta = r\dot{\theta}$$

Ans.

$$\boxed{\underset{\sim}{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\,\underset{\sim}{e_r} + (r\ddot{\theta} + \cancel{2\dot{r}\dot{\theta}})\,\underset{\sim}{e_\theta}} \longrightarrow a_r = -r\dot{\theta}^2, \quad a_\theta = r\ddot{\theta}$$

Ans.