Dynamics of Particle

1. Introduction to Dynamics : Newton's Laws, units, dimensions.
2. Kinematics of Particles
3. Kinetics of Particles
4. Kinetics of Systems of Particles

Dynamics of Rigid Bodies

1. Plane Kinematics of Rigid Bodies
2. Plane Kinetics of Rigid Bodies
3. 3D Dynamics of Rigid Bodies
4. Vibrations

Chapter 2. Kinematics of Particle

Rectilinear Motion

Plane Curvilinear Motion

- Rectangular coordinate (x-y)
- Normal-Tangent Coordinate (n-t)
- Polar coordinate (r-θ)

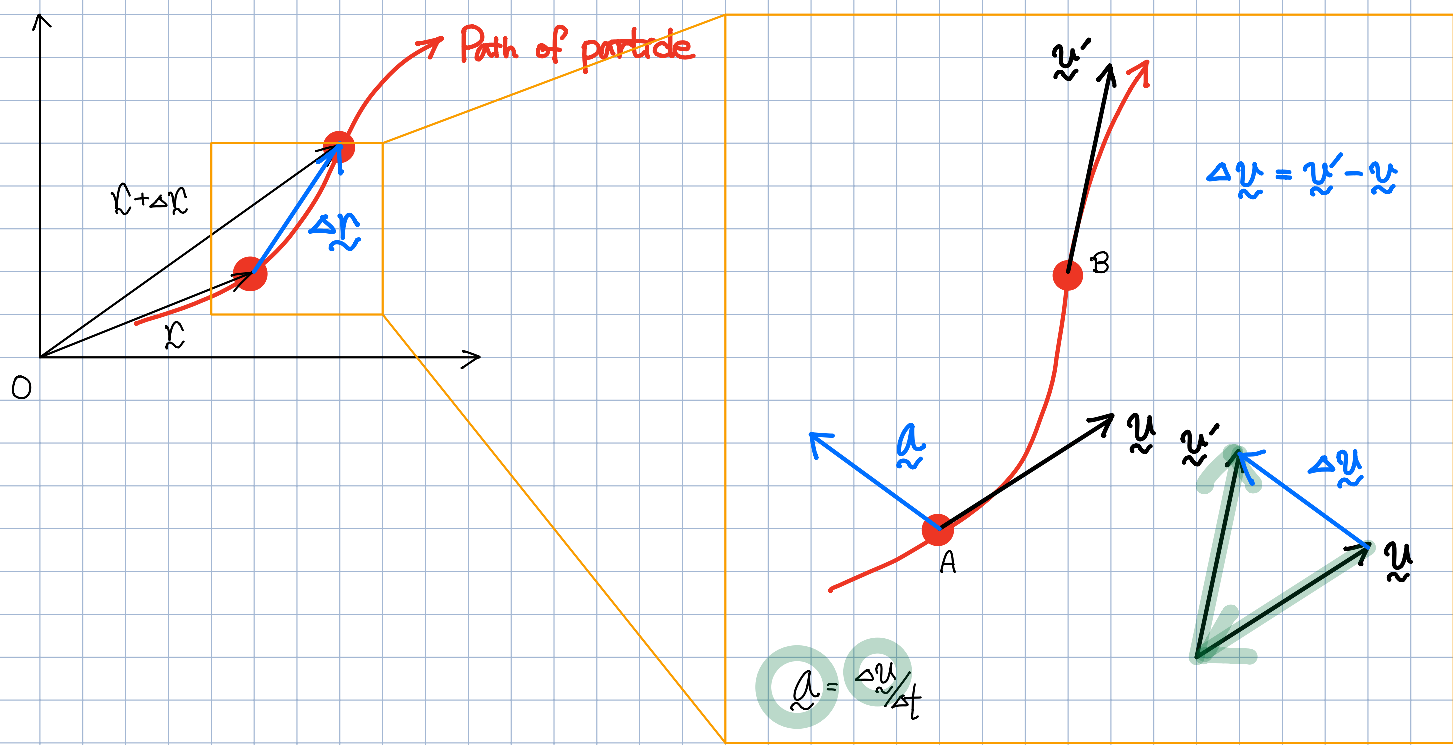
Space Curvilinear Motion

- Rectangular coordinate (x-y-z)
- Cylindrical coordinate (r-θ-z)
- Spherical coordinate (R-θ-ϕ)

Constrained Motion of Connected Particles

Plane Curvilinear Motion (Curve + Linear Motion)

: The motion of an object moving in a curved path
It combines straight sections of motion with curved sections



The instantaneous velocity

$$\boxed{\begin{aligned}\tilde{v} &= \lim_{\Delta t \to 0} \Delta\tilde{r}/\Delta t \\ &= d\tilde{r}/dt = \dot{\tilde{r}}\end{aligned}} \longleftrightarrow \boxed{\begin{aligned}&s \\ &v = ds/dt \\ &a = dv/dt\end{aligned}}$$

Rectilinear Motion.

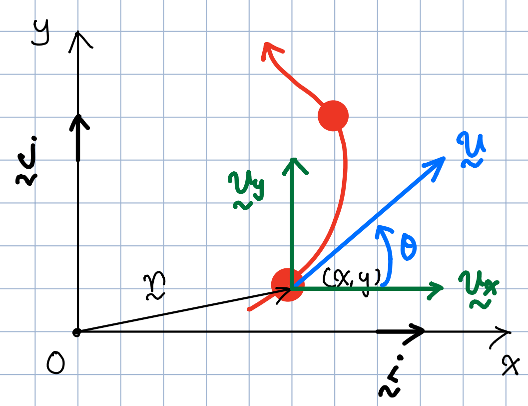
the time derivative of a scalar

Plane curvilinear Motion

the time derivative of a vector

$$\boxed{\begin{aligned}\tilde{a} &= \lim_{\Delta t \to 0} \Delta\tilde{v}/\Delta t \\ &= d\tilde{v}/dt = \dot{\tilde{v}}\end{aligned}}$$

Rectangular Coordinates (x-y)



$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d}{dt}(x\,\mathbf{i} + y\,\mathbf{j})$$

$$= (\dot{x}\,\mathbf{i} + \cancel{x\,\dot{\mathbf{i}}}) + (\dot{y}\,\mathbf{j} + \cancel{y\,\dot{\mathbf{j}}})$$

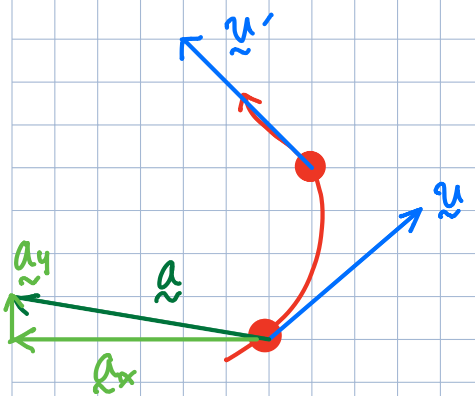
$$= \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d}{dt}(\dot{\mathbf{r}})$$

$$= \frac{d}{dt}(\dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j})$$

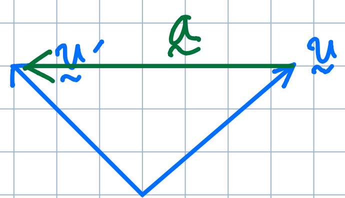
$$= (\ddot{x}\,\mathbf{i} + \cancel{\dot{x}\,\dot{\mathbf{i}}}) + (\ddot{y}\,\mathbf{j} + \cancel{\dot{y}\,\dot{\mathbf{j}}})$$

$$= \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j}$$



$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{\Delta \mathbf{v}}{\Delta t}$$

$$\Delta \mathbf{v} = \mathbf{v}' - \mathbf{v}$$



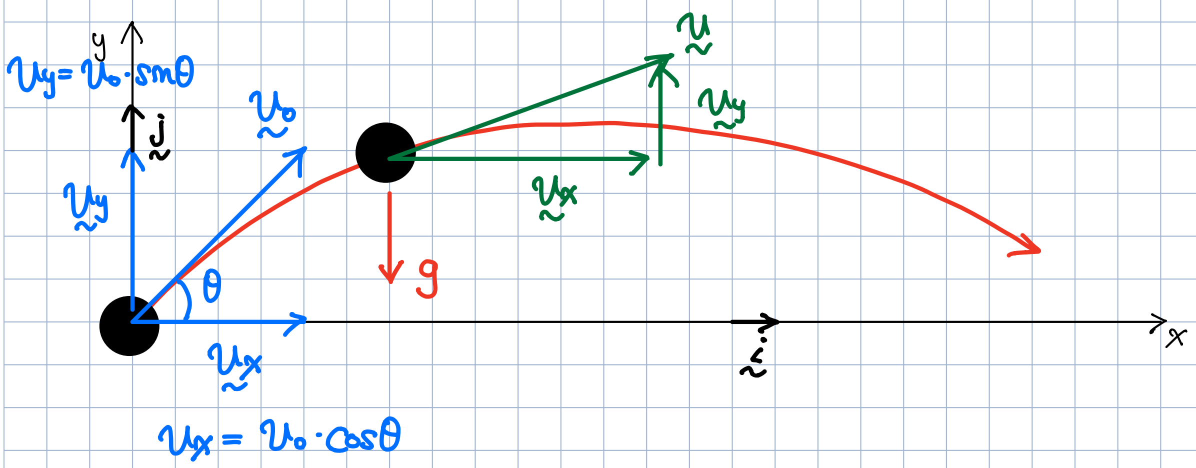
$$v^2 = v_x^2 + v_y^2 \qquad a^2 = a_x^2 + a_y^2$$

$$v = \sqrt{v_x^2 + v_y^2} \qquad a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{v_y}{v_x}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Example > Projectile Motion



$$\underset{\sim}{a} = g(-\underset{\sim}{j}) = -g\underset{\sim}{j}$$

$$a_x = 0, \quad a_y = -g$$

$$\boxed{\begin{aligned} &s \\ &v = ds/dt \\ &a = dv/dt \end{aligned}} \longrightarrow a\,dt = dv$$

$$\int_0^t a\,dt = \int_{v_0}^{v} dv$$

$$= a \cdot \int_0^t dt$$

$$= a \cdot t = v - v_0$$

$$v = v_0 + at$$

x 방향 : $v_x = (v_x)_0 + \cancel{a_x} \cdot t$

$v_x = (v_x)_0$ Ans.

y 방향 : $v_y = (v_y)_0 + a_y t$

$v_y = (v_y)_0 - gt$ Ans.

$$v = ds/dt \longrightarrow v\,dt = ds$$

$$\int_0^t v\,dt = \int_{s_0}^{s} ds$$

x방향 : $v_x = (v_x)_0$

$$v \cdot \int_0^t dt = \int_{s_0}^{s} ds$$

$$v \cdot (t - 0) = s - s_0$$

$$(v_x)_0 \cdot t = x - x_0$$

$$\therefore x = x_0 + (v_x)_0 \cdot t$$

y방향 : $v_y = (v_y)_0 - gt$

$$\int_0^t v_y\,dt = \int_{s_0}^{s} ds$$

$$\int_0^t \{(v_y)_0 - gt\}\,dt = s - s_0$$

$$(v_y)_0 t - \frac{1}{2}gt^2 = y - y_0$$

$$\therefore y = y_0 + (v_y)_0 \cdot t - \frac{1}{2}gt^2 \quad \text{Ans}$$