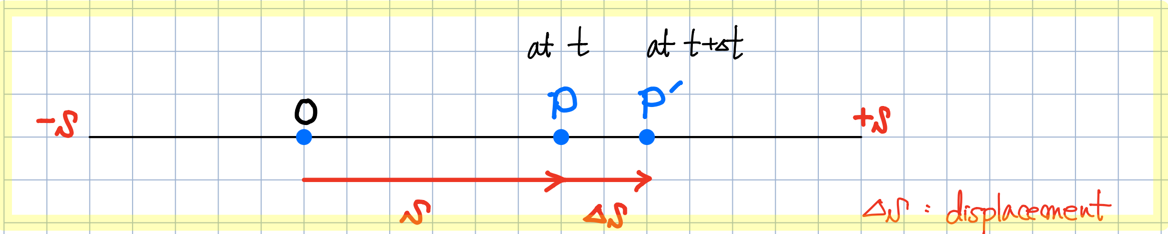
Chapter 2. Kinematics of Particles

Rectilinear Motion



a particle, P, is moving along a straight line.

velocity & acceleration

- **velocity** : the time rate of change of the position coordinate s.

$\underbrace{\text{change of the position coordinate } s}_{\Delta s}$

the average velocity : $v_{av} = \Delta s / \Delta t$

the instantaneous velocity : $v = \lim_{\Delta t \to 0} \Delta s / \Delta t = ds/dt = \dot{s}$

- **acceleration**

the average acceleration : $a_{av} = \Delta v / \Delta t$

the instantaneous acceleration : $a = \lim_{\Delta t \to 0} \Delta v / \Delta t = dv/dt = \dot{v}$

$a = dv/dt = \frac{d}{dt}(v) \leftarrow v = ds/dt$

$= \frac{d}{dt}\left(\frac{ds}{dt}\right)$

$= \frac{d^2 s}{dt^2} = \ddot{s}$

$\boxed{v = ds/dt}$ $\boxed{a = dv/dt}$

$dt = dv/a$

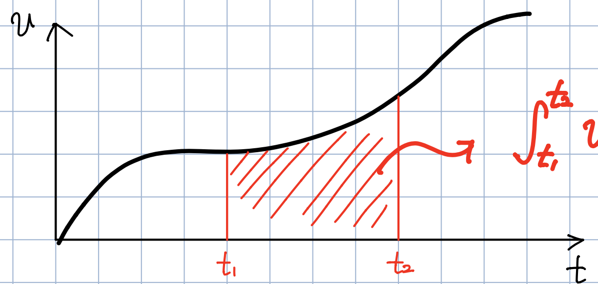
$v = ds/(dv/a) = a\,ds/dv \longrightarrow$ $v\,dv = a\,ds$ $\longleftarrow v = \dot{s}$, $a = \dot{v} = \ddot{s}$

$\dot{s}\,d\dot{s} = \ddot{s}\,ds$

$$\boxed{\begin{array}{l} s \\ v = ds/dt \\ a = dv/dt \end{array}} \longrightarrow \quad ds = v\,dt$$

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt$$

$$\Delta s = s_2 - s_1 = \int_{t_1}^{t_2} v(t)\,dt \qquad \text{(area under } v\text{-}t \text{ curve)}$$



$$dv = a\,dt$$

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

$$v\Big]_{v_1}^{v_2} = v_2 - v_1 = \int_{t_1}^{t_2} a(t)\,dt \qquad \text{(area under } a\text{-}t \text{ curve)}$$

$$\boxed{v\,dv = a\,ds} \longleftarrow \boxed{\begin{array}{c} v = ds/dt \quad , \quad a = dv/dt \\ dt = ds/v \;=\; dt = dv/a \\ ds/v = dv/a \\ v\,dv = a\,ds \end{array}}$$

$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds$$

$$\frac{1}{2}v^2\Big]_{v_1}^{v_2} = \frac{1}{2}(v_2^2 - v_1^2) = \text{area under } a\text{-}s \text{ curve.}$$

$$\boxed{\begin{array}{l} s \\ v = ds/dt \\ a = dv/dt \\ v\,dv = a\,ds \end{array}}$$

const a :

$a = dv/dt$

$dv = a\,dt$

$\int_{v_0}^{v} dv = \int_0^t a\,dt = a \cdot \int_0^t dt$

$v - v_0 = at$

$\underline{v = v_0 + at}$ **Ans.**

$ds = v\,dt$

$\int_{s_0}^{s} ds = \int_0^t v\,dt$

$= \int_0^t (v_0 + at)\,dt$

$s - s_0 = v_0 t + \frac{1}{2}at^2$

$\underline{s = s_0 + v_0 t + \frac{1}{2}at^2}$ **Ans.**

$v\,dv = a\,ds$

$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a\,ds = a \cdot \int_{s_0}^{s} ds$

$\frac{1}{2}v^2\Big]_{v_0}^{v} = a \cdot (s - s_0)$

$\frac{1}{2}(v^2 - v_0^2) = a \cdot (s - s_0)$

$v^2 - v_0^2 = 2a(s - s_0)$

$\underline{v^2 = v_0^2 + 2a(s - s_0)}$ **Ans.**

Example > 2.1

a particle : move along a straight line

position : $s = 2t^3 - 24t + 6$ [m] , t [sec]

a) $v = 72\,m/s \longrightarrow t = ?$ ($t_0 = 0\,sec$)

b) $v = 30\,m/s \longrightarrow a = ?$

c) the net displacement , Δs ($t = 1 \sim 4\,sec$)

a) $\boxed{v = \frac{ds}{dt}} \leftarrow s = 2t^3 - 24t + 6$

$= \frac{d}{dt}(2t^3 - 24t + 6)$

$= 6t^2 - 24$

$v = 6t^2 - 24 = 72$

$6t^2 = 96$

$t^2 = 16$

$t = 4\,sec$ Ans.

b) $\boxed{a = \frac{dv}{dt}} \leftarrow v = 6t^2 - 24$

$= \frac{d}{dt}(6t^2 - 24)$

$= 12t = 36\,m/s^2$ Ans.

$v = 30 = 6t^2 - 24$

$6t^2 = 54$

$t^2 = 9$

$t = 3\,sec$

c) $\Delta s = s_{t=4sec} - s_{t=1sec}$

$= \{2 \cdot 4^3 - 24 \cdot 4 + 6\} - \{2 \cdot 1^3 - 24 \cdot 1 + 6\}$

$= \{2 \cdot 64 - 96 + 6\} - \{2 - 24 + 6\}$

$= \{128 - 90\} - \{-16\}$

$= 38 + 16$

$= 54\,m$ Ans.

Example > 2.2

A particle : move along the x-axis

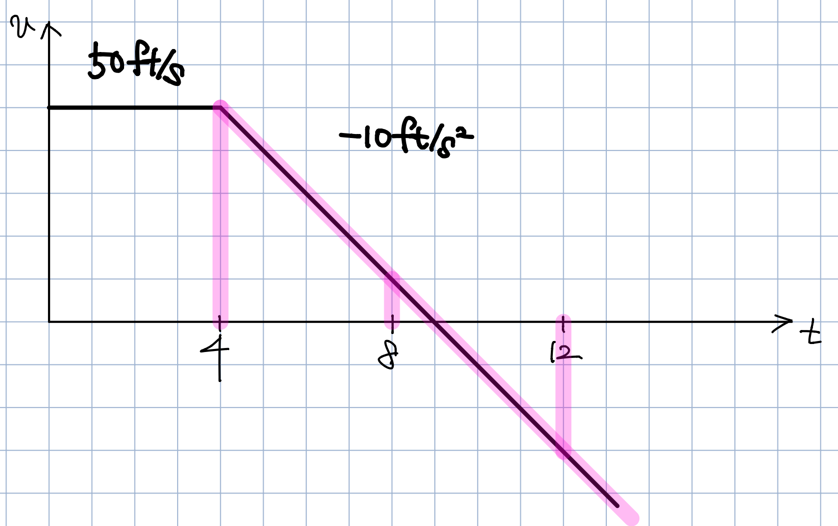
$v_x = 50$ ft/sec (initial velocity)

$t = 0-4$sec : $v_x = 50$ ft/sec ($a_x = 0$)

$t = 4$sec : $a_x = -10$ ft/s^2

a) Determine the velocity, x-coordinate of a particle when $t = 8$sec, 12sec.

b) Find the maximum positive x-coordinate.



t = 8sec

$$\boxed{a = dv/dt}$$

$a\,dt = dv$

$$\int_{t_1}^{t_2} a\,dt = \int_{v_1}^{v_2} dv \quad \leftarrow \quad a = -10 \text{ft/s}^2 \text{ (const)}$$

$$a \cdot (t_2 - t_1) = v_2 - v_1 \quad \leftarrow \quad t_1 = 4\text{sec}, \; t_2 = 8\text{sec}$$

$(-10) \cdot (8-4) = v_2 - 50$

$-40 = v_2 - 50$

$v_2 = 10$ ft/s Ans.

t = 12sec

$$a \cdot (t_2 - t_1) = v_2 - v_1 \quad \leftarrow \quad t_1 = 4, \; t_2 = 12$$

$(-10) \cdot (12-4) = v_2 - 50$

$-80 = v_2 - 50$

$\therefore v_2 = -30$ ft/s Ans.

$\underline{t = 8\,sec}$

$$\boxed{v = ds/dt} \longrightarrow ds = v\,dt.$$

$$\int_{s_0}^{s} ds = \int_0^t v\,dt. \longleftarrow v = 50 - 10t$$

$$s - s_0 = \int_0^t (50 - 10t)\,dt$$

$$= 50t - 5t^2$$

$$\therefore s = s_0 + 50t - 5t^2$$

$$\boxed{a = dv/dt}$$

$$dv = a\,dt$$

$$\int_{v_0}^{v} dv = \int_0^t a\,dt$$

$$v - v_0 = at \longleftarrow a = -10\,ft/s^2,\ v_0 = 50\,ft/sec$$

$$v = 50 - 10t$$

$\underline{t = 8\,sec}$

$$s = s_0 + 50\cdot(4) - 5(4)^2 \longleftarrow s_0$$

$$= (200\,ft) + 200 - 80$$

$$= 320\,ft \quad Ans.$$

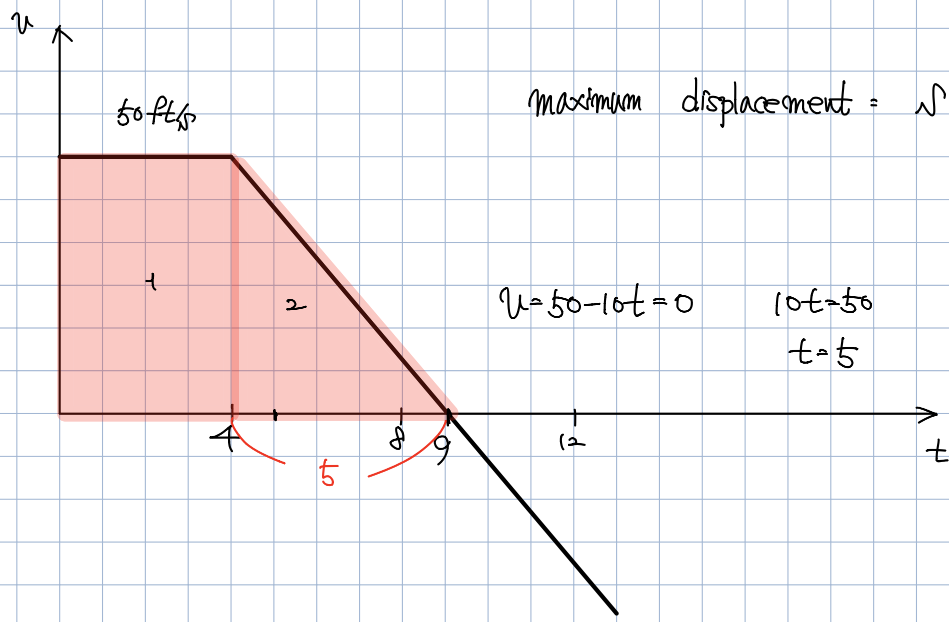
$\underline{t = 12\,sec}$

$$s = s_0 + 50\cdot(8) - 5(8)^2 \longleftarrow s_0 = (50\,ft/s)\cdot(4\,s)$$

$$= 200\,ft$$

$$= (200) + (400) - 320$$

$$= 280\,ft \quad Ans.$$



$$v = 50 - 10t = 0 \qquad 10t = 50$$

$$t = 5$$

maximum displacement : $s = (50 \times 4) + (50 \times 5 \times 1/2)$

$$= 200 + 125$$

$$= 325\,ft \quad Ans.$$