자동제어 한방에 끝내기

7.State Space Analysis

- Find the steady-state value for x for a step input u, assuming zero IC

$$\dot{x} = \begin{bmatrix} -4 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$sX = \underbrace{\begin{bmatrix} -4 & 1 \\ -2 & -1 \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} U \longrightarrow (sI - A)X = BU$$

$$X = (sI-A)^{-1} BU$$

$$= \begin{bmatrix} s+4 & -1 \\ 2 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$\therefore X = \frac{1}{(s+4)(s+1) - (-1)(2)} \begin{bmatrix} s+1 & 1 \\ -2 & s+4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$= \frac{1}{s^2+5s+4+2} \begin{bmatrix} 1 \\ s+4 \end{bmatrix} U$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{s^2+5s+6} \begin{bmatrix} 1 \\ s+4 \end{bmatrix} \frac{1}{s}$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = s\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \lim_{s\to 0}\left\{ s \cdot \frac{1}{s} \cdot \frac{1}{s^2+5s+6} \begin{bmatrix} 1 \\ s+4 \end{bmatrix} \right\}$$

$$= \lim_{s\to 0} \frac{1}{s^2+5s+6} \begin{bmatrix} 1 \\ s+4 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\lim_{t\to\infty} x_1(t) = \frac{1}{6}, \quad \lim_{t\to\infty} x_2(t) = \frac{2}{3}$$

- Design a state feedback controller that satisfies the following specifications
  - Closed-loop poles having $\zeta = \frac{1}{\sqrt{2}}$ $u = -Kx$
  - $t_p \leq \pi \rightarrow \frac{\pi}{\omega_d} \leq \pi, \therefore \omega_d \geq 1, \therefore \omega_n\sqrt{1-\zeta^2} \geq 1 \longrightarrow \omega_n \geq \sqrt{2}$

$$\dot{x} = \underset{A}{\begin{bmatrix} 0 & 1 \\ 6 & 5 \end{bmatrix}} x + \underset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u$$

$$u = -Kx = -K\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\begin{bmatrix} k_1 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = Ax - BKx$$

$$= (A - BK)x$$

$$sX = (A - BK)X, \quad \therefore (sI - A + BK)X = 0$$

$$BK = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$$

char. eq. $|sI - A + BK| = \begin{vmatrix} s & -1 \\ -6+k_1 & s-5+k_2 \end{vmatrix} = s(s-5+k_2) + (k_1 - 6)$

$$= s^2 + (-5+k_2)s + (k_1 - 6)$$

$$\boxed{k_1 = 8}$$

$$\boxed{k_2 = 7}$$

$$u = \begin{bmatrix} 8 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

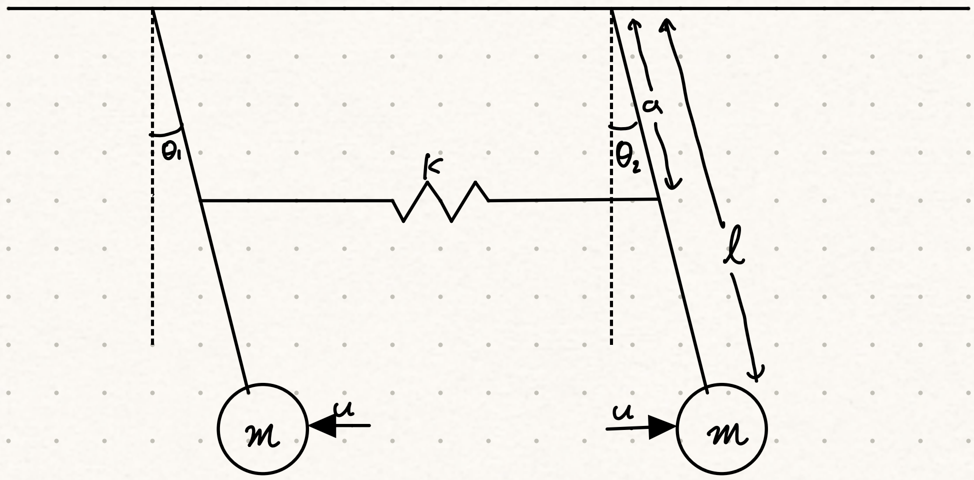
- Two pendulums coupled by a string is controlled by two equal and opposite forces u, as shown in the figure.

Eq. of motion is

$$ml^2\ddot{\theta}_1 = -ka^2(\theta_1 - \theta_2) - mgl\theta_1 - lu$$
$$ml^2\ddot{\theta}_2 = -ka^2(\theta_2 - \theta_1) - mgl\theta_2 + lu$$

(a) Is the system controllable? What is the physical meaning?

(b) How can we make the system controllable?



$$ml^2\ddot{\theta}_1 = -ka^2(\theta_1 - \theta_2) - mgl\theta_1 - lu$$
$$ml^2\ddot{\theta}_2 = -ka^2(\theta_2 - \theta_1) - mgl\theta_2 + lu$$

$$\ddot{\theta}_1 = -\frac{ka^2}{ml^2}\theta_1 - \frac{g}{l}\theta_1 + \frac{ka^2}{ml^2}\theta_2 - \frac{1}{ml}u$$

$$\frac{d}{dt}\begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{ka^2}{ml^2} + \frac{g}{l}\right) & 0 & \frac{ka^2}{ml^2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{ka^2}{ml^2} & 0 & -\left(\frac{ka^2}{ml^2} + \frac{g}{l}\right) & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{ml} \\ 0 \\ \frac{1}{ml} \end{bmatrix} u$$

$$C = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{ml} & 0 & \left(2\frac{ka^2}{ml^2} + \frac{g}{l}\right)\frac{1}{ml} \\ -\frac{1}{ml} & 0 & \left(2\frac{ka^2}{ml^2} + \frac{g}{l}\right)\frac{1}{ml} & 0 \\ 0 & \frac{1}{ml} & 0 & -2\left(\frac{ka^2}{ml^2} + \frac{g}{l}\right)\frac{1}{ml} \\ \frac{1}{ml} & 0 & -\left(2\frac{ka^2}{ml^2} + \frac{g}{l}\right)\frac{1}{ml} & 0 \end{bmatrix}$$

(a) Un Controllable

$$ml^2\ddot{\theta}_1 = -ka^2(\theta_1 - \theta_2) - mgl\theta_1 - lu$$
$$ml^2\ddot{\theta}_2 = -ka^2(\theta_2 - \theta_1) - mgl\theta_2 + lu$$

$$ml^2(\ddot{\theta}_1 + \ddot{\theta}_2) = -mgl(\theta_1 + \theta_2)$$

$$ml^2(\ddot{\theta}_1 - \ddot{\theta}_2) = -2ka^2(\theta_1 - \theta_2) - mgl(\theta_1 - \theta_2) - 2lu$$

(b) u 를 대칭 X, $\boxed{u_1, u_2}$