자동제어 한방에 끝내기

7.State Space Analysis

목차

State

- Why?
- SISO / MIMO systems
- Review of Linear Algebra

State Equations

- Controllability & Observability
- Canonical Forms
- State Feedback
- Linear Quadratic Regulator

State Feedback

- State에 기반한 자동제어 시스템을 만든다고 가정
- Input $u = \overset{-Kx}{Kx}$를 가정하고, 시스템에 맞도록 K를 설계
- Controllable, Observable 가정

$K \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$ $x : \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\dot{x} = Ax + Bu$$

$$= Ax - BKx$$

$$\dot{x} = (A - BK)x$$

State Feedback

$$\dot{x} = Ax + Bu$$

$$\dot{x} = Ax - BKx = (A - BK)x \Rightarrow sX - x(0) = (A - BK)X$$

$$\Rightarrow x(t) = e^{(A-BK)t}x(0)$$

$$sIX - x(0) = \underbrace{(A - BK)}_{A_{cl}}X$$

$$(sI - A_{cl})X(s) = x(0), \therefore X(s) = \frac{(sI - A_{cl})^{-1}}{}x(0)$$

$$\frac{[\;\;]}{|sI - \boxed{A_{cl}}|} \mapsto p(s) \Rightarrow (s+1)(s^2+2s+3)$$

Linear Quadratic Regulator

- **이상적인** 제어 이득을 결정
- **State**와 **input** 모두를 고려하여 제어 이득을 결정
- $J = \min \int_0^{\infty} (x^T Q x + u^T R u)\, dt$ 값을 최소화하는 K값의 결정 (Q ≥ 0, R > 0)

Ru^2

$$\begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix}^T \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\boxed{u = -Kx}$$

Linear Quadratic Regulator

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt \qquad Q \geq 0,\quad R > 0,\quad \boxed{P \geq 0}$$

$$= x_0^T P x_0 \boxed{- x_0^T P x_0} + \int_0^\infty (x^T Q x + u^T R u)\, dt$$

$$\int_0^\infty \frac{d}{dt}\left[x^T P x\right] dt \rightarrow x_\infty^T P x_\infty - x_0^T P x_0$$

$$= x_0^T P x_0 + \int_0^\infty \left(\frac{d}{dt}\left[x^T P x\right] + x^T Q x + u^T R u\right) dt$$

$$= x_0^T P x_0 + \int_0^\infty \left\{(Ax + Bu)^T P x + x^T P (Ax + Bu) + x^T Q x + u^T R u\right\} dt$$

$$\rightarrow x^T A^T P x + u^T B^T P x + x^T P A x + x^T P B u + x^T Q x + u^T R u$$

$$= x^T (A^T P + PA + Q) x + u^T B^T P x + x^T P B u + u^T R u$$

$$u^T R u + u^T B P x + x^T P B u + x^T (A^T P + PA + Q) x$$

$$\boxed{(u + R^{-1} B P x)^T R (u + R^{-1} B P x) + x^T (A^T P + PA + Q - P B^T R^{-1} B P) x}$$

u, P

$$u^T R u + u^T B P x + x^T P B u + x^T P B^T R^{-1} B P x - x^T P B^T R^{-1} B P x + x^T (A^T P + PA + Q) x$$

$$\boxed{\therefore u = -R^{-1} B P x}$$

$$\boxed{A^T P + PA + Q - P B^T R^{-1} B P = 0} \ , P$$

CARE