자동제어 한방에 끝내기

7.State Space Analysis

목차

State

- Why?
- SISO / MIMO systems
- Review of Linear Algebra

State Equations

- Controllability & Observability
- Canonical Forms
- State Feedback
- Linear Quadratic Regulator

State systems

$$\dot{x} = A x + Bu$$
$$y = Cx + Du$$

$\boxed{\hat{x} = Px}$

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}-6 & -5\\ 1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ x\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u$$

$\dot{\hat{x}} = P\dot{x}$

$P\dot{x} = P(Ax + Bu)$

$= PA \cdot x + PBu$

$\dot{\hat{x}} = \underbrace{PAP^{-1}}_{\hat{A}}\hat{x} + \underbrace{PB}_{\hat{B}}u \Rightarrow \underline{\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u}$

$C = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$

$\hat{C} = \begin{bmatrix} \hat{B} & \hat{A}\hat{B} & \hat{A}^2\hat{B} & \cdots & \hat{A}^{n-1}\hat{B} \end{bmatrix}$

$= \begin{bmatrix} PB & PAB & PA^2B & \cdots & PA^{n-1}B \end{bmatrix}$

$= PC$

Canonical forms

- 같은 TF를 나타낼 때, 상태변수를 무엇으로 잡느냐에 따라서 State-space representation이 달라질 수 있다.
- Control Canonical form
- Modal Canonical form
- Observer Canonical form

Canonical forms

- Control Canonical form

$$\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}\,\frac{\boldsymbol{z}(\boldsymbol{s})}{\boldsymbol{z}(\boldsymbol{s})}$$

$$y = b_1 \ddot{z} + b_2 \dot{z} + b_3 z \;\Big/\; u = \dddot{z} + a_1 \ddot{z} + a_2 \dot{z} + a_3 z$$

$$x_1 = z,\; x_2 = \dot{z},\; x_3 = \ddot{z}$$

$$\rightarrow \dddot{z} = -a_1 \ddot{z} - a_2 \dot{z} - a_3 z + u$$

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dddot{z} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}} \begin{bmatrix} x_1 = z \\ x_2 = \dot{z} \\ x_3 = \ddot{z} \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} u$$

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & * \\ 1 & -a_1 & * \end{bmatrix}$$

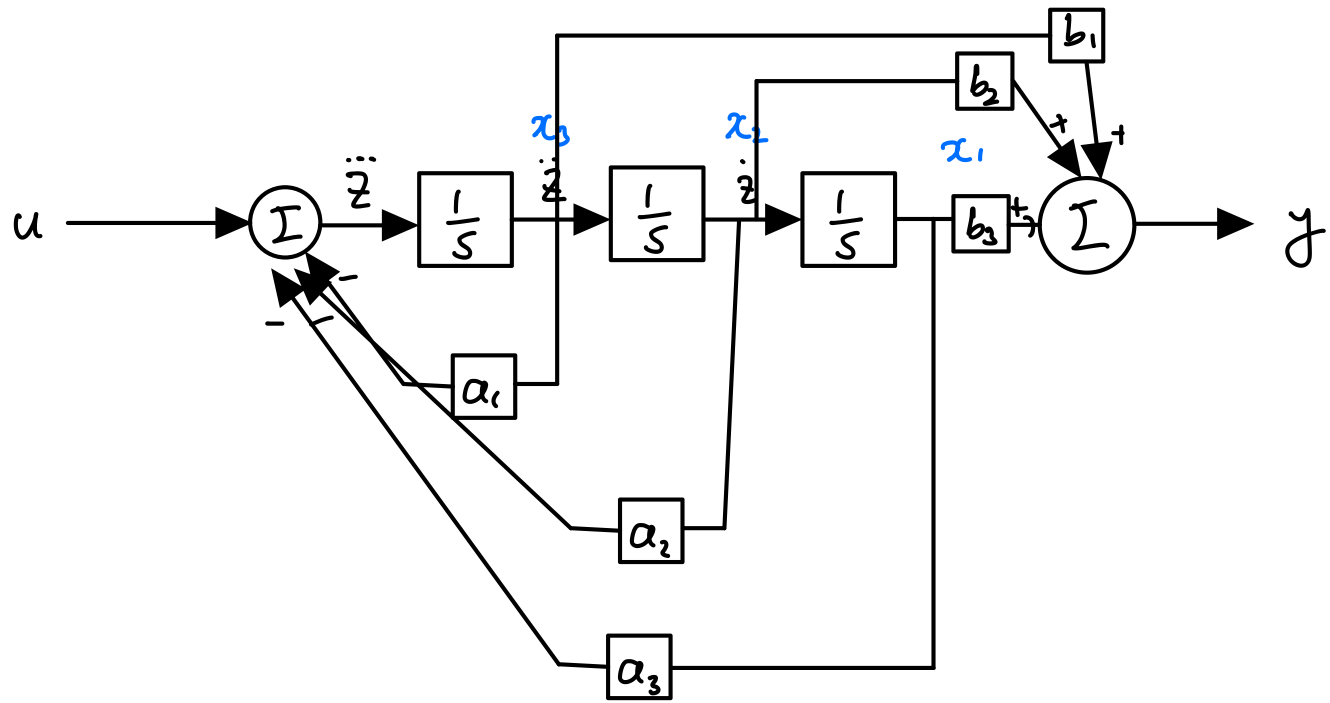
$$y = \overset{C}{\begin{bmatrix} b_3 & b_2 & b_1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \overset{D}{\begin{bmatrix} 0 \end{bmatrix}} u$$

Canonical forms

- Control Canonical form

$$y = b_1\ddot{z} + b_2\dot{z} + b_3 z \;/\; u = \dddot{z} + a_1\ddot{z} + a_2\dot{z} + a_3 z$$

$$u - a_1\ddot{z} - a_2\dot{z} - a_3 z = \dddot{z}$$



Canonical forms

- Observer Canonical form

$$\frac{Y(s)}{U(s)} \Rightarrow \dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = b_1\ddot{u} + b_2\dot{u} + b_3 u$$

$$\dddot{y} = \left(b_1\ddot{u} - a_1\ddot{y}\right) + \left(b_2\dot{u} - a_2\dot{y}\right) + \left(b_3 u - a_3 y\right)$$

$y = x_1$

$$\dot{y} = b_1 u - a_1 y + \underbrace{\int (b_2 u - a_2 y)\,dt + \iint (b_3 u - a_3 y)\,dt^2}_{x_2} \Rightarrow \dot{x}_1 = b_1 u - a_1 x_1 + x_2$$

$$\dot{x}_2 = b_2 u - a_2 x_1 + \underbrace{\int (b_3 u - a_3 y)\,dt}_{x_3} \Rightarrow \dot{x}_2 = b_2 u - a_2 x_1 + x_3$$

$$\underline{\dot{x}_3 = b_3 u - a_3 x_1}$$

Canonical forms

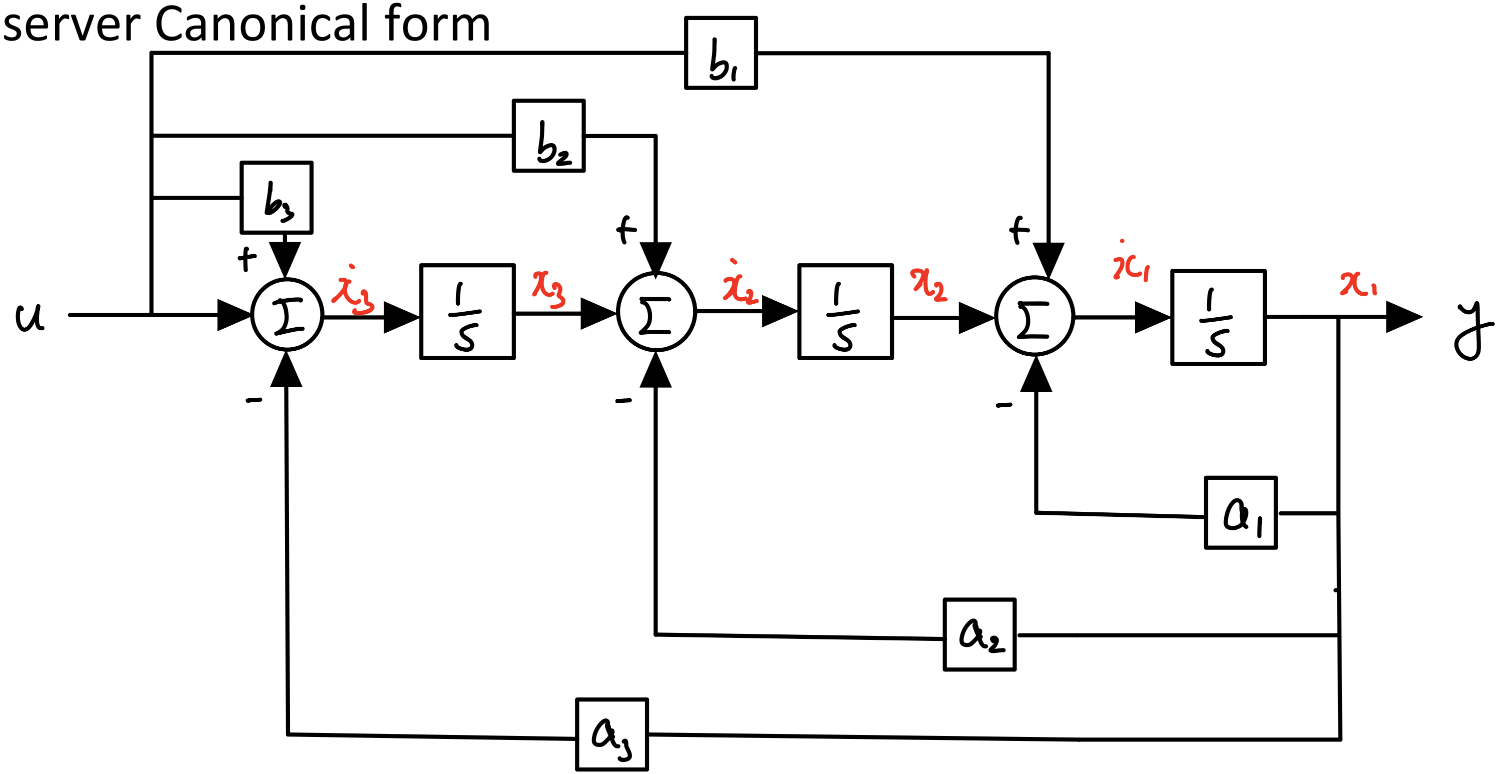
- Observer Canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0 \quad u$$

Canonical forms

- Observer Canonical form



Canonical forms

- Modal Canonical form
  - **eigenvalues**

$$\frac{Y(s)}{U(s)} = \underline{\frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \frac{k_3}{s+p_3}} \rightarrow Y(s) = k_1 \underbrace{\left[\frac{U(s)}{s+p_1}\right]}_{x_1} + k_2 \underbrace{\left[\frac{U(s)}{s+p_2}\right]}_{x_2} + k_3 \underbrace{\left[\frac{U(s)}{s+p_3}\right]}_{x_3}$$

$$\frac{U(s)}{s+p_1} = x_1(s) \Rightarrow \dot{x}_1 = -p_1 x_1 + u$$

$$x_2(s) \Rightarrow \dot{x}_2 = -p_2 x_2 + u$$

$$\dot{x}_3 = -p_3 x_3 + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -p_1 & & \\ & -p_2 & \\ & & -p_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0\,u$$