자동제어 한방에 끝내기

6.Frequency Response

목차

Frequency domain

- Why?
- System에 진동하는 input을 넣으면?

System & Stability analysis

- Bode plot
- Stability margin
- Nyquist plot

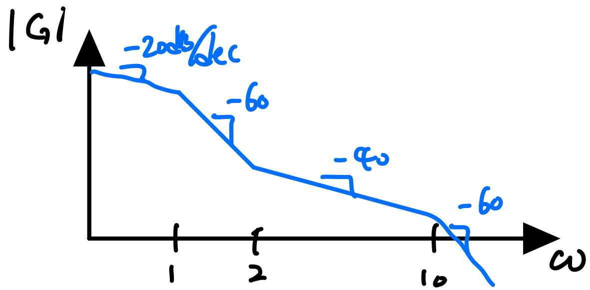
Dynamic compensation

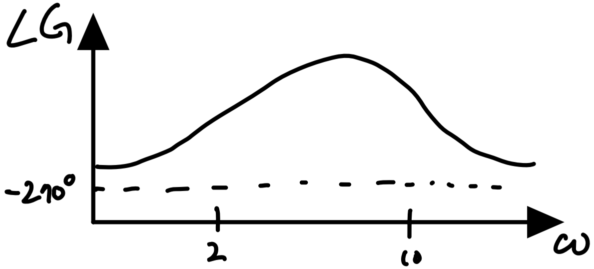
- Lead compensation
- Lag compensation

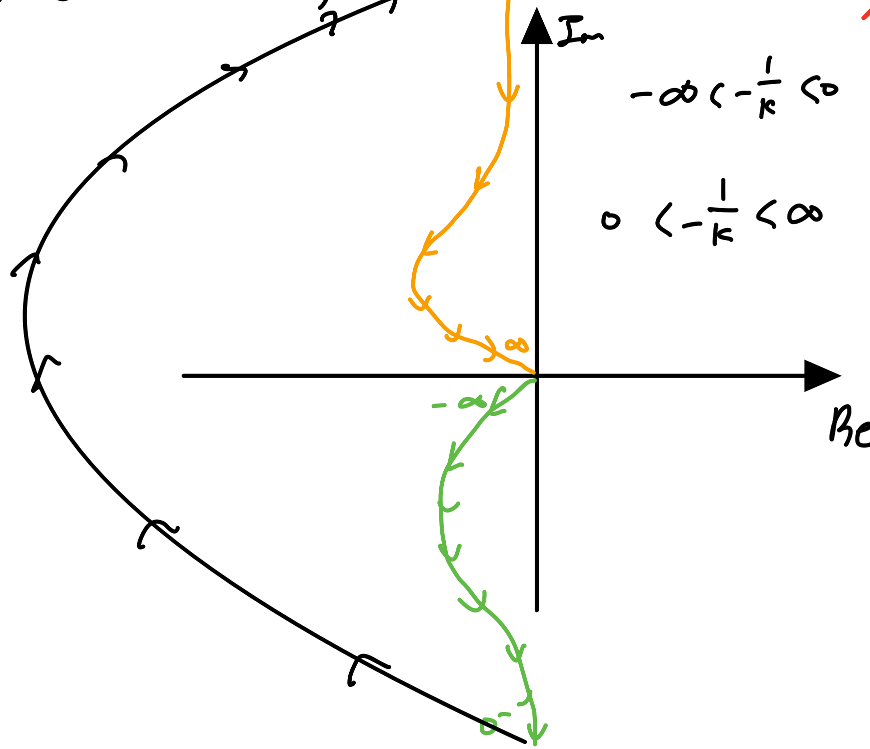
문제풀이

$K\dfrac{s+2}{s(s+10)(s^2-1)} \sim \dfrac{1}{\varepsilon e^{i\theta}} \cdot -1$, $\quad -1 \to e^{i\pi}$ $\quad = \dfrac{1}{\varepsilon} \cdot e^{i(\pi-\theta)}$

$S = \varepsilon e^{i\theta} \; \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$







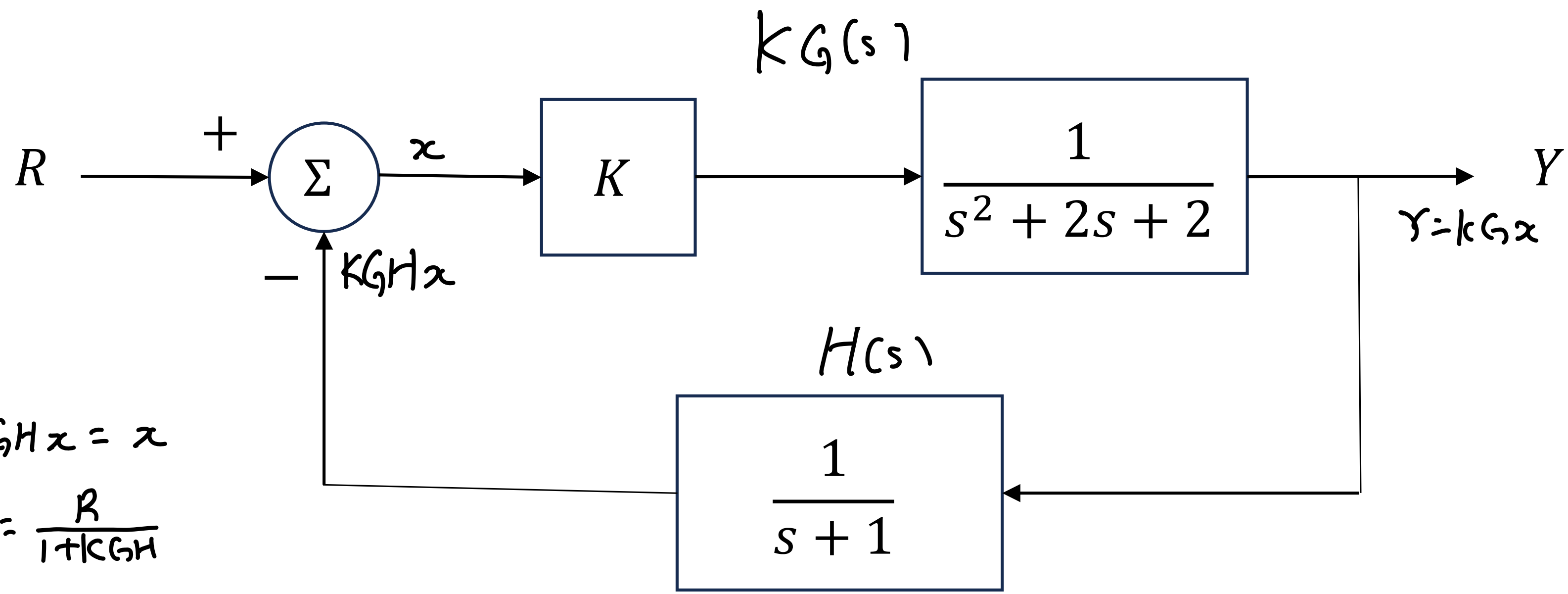
$N = Z - P$

$-\infty < -\frac{1}{K} < 0 \qquad 1 = \boxed{2} - 1$

$0 < -\frac{1}{K} < \infty \qquad 0 = 1 - 1$

K가 어떤 범위에든 CLTF 불안정하다.

문제풀이



$$R - KGHx = x$$

$$x = \frac{R}{1+KGH}$$

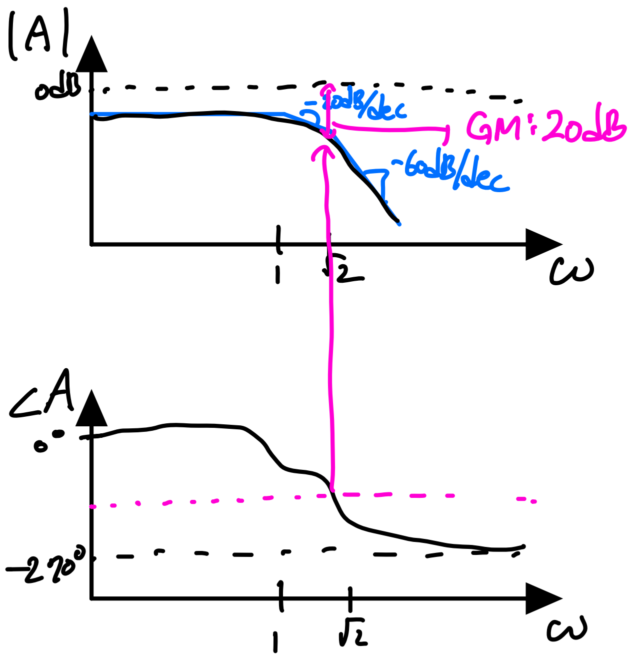
$$\therefore Y = \frac{KG}{1+KGH} \cdot R \quad \Rightarrow \text{Closed-loop characteristic function}$$

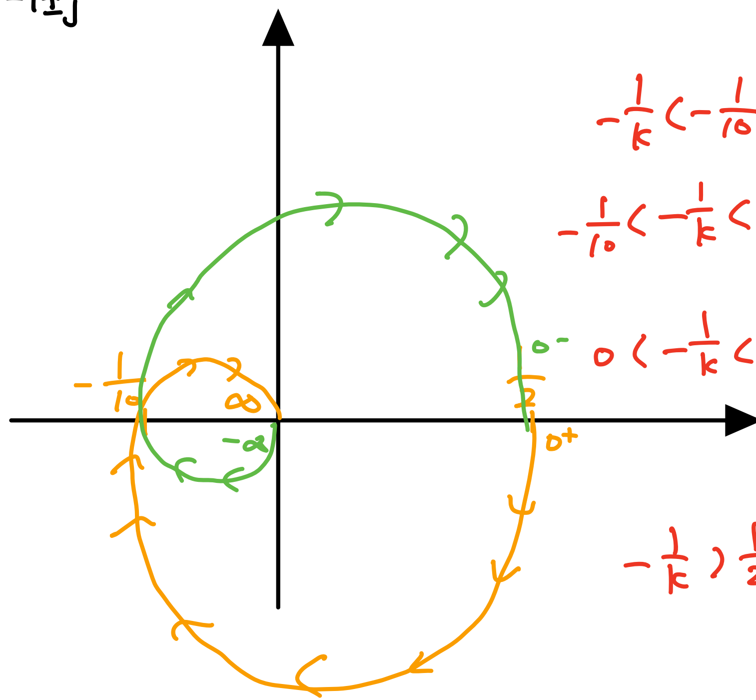
$$1 + K \cdot \frac{1}{(s+1)(s^2+2s+2)}$$

문제풀이

$A(s) = \frac{1}{(s+1)(s^2+2s+2)}$ → zero X, → pole : -1, $-1 \pm j$

$N = Z - \underset{0}{P}$





$-\frac{1}{k} < -\frac{1}{10}$ → $N=0$, ∴ $Z=0$
stable

$-\frac{1}{10} < -\frac{1}{k} < 0$ → $N=2$, ∴ $Z=2$
2 RHP CL pole

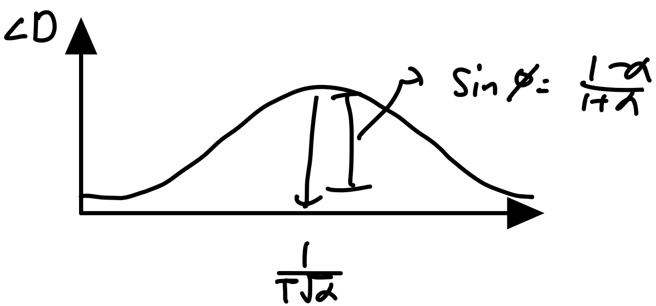
$0 < -\frac{1}{k} < \frac{1}{2}$ → $N=1$, ∴ $Z=2$
1 RHP CL pole

$-\frac{1}{k} > \frac{1}{2}$ → $N=0$, ∴ $Z=0$
stable

CL stable k → $-2 < k < 10$

문제풀이

For the Lead compensator $D(s)=\frac{Ts+1}{\alpha Ts+1}$, show that the frequency where the phase is maximum is given by $\omega_{max}=\frac{1}{T\sqrt{\alpha}}$, and the corresponding $\sin\phi_{max}=\frac{1-\alpha}{1+\alpha}$



$$\frac{d}{d\omega}(\angle D)\Big|_{\omega_{max}}=0$$

$$\angle D(j\omega) = \frac{\angle(1+\omega Tj)}{\angle(1+\omega\alpha Tj)} = \angle(1+\omega Tj) - \angle(1+\omega\alpha Tj)$$

$$\tan^{-1}(\omega T) - \tan^{-1}(\omega\alpha T) = 0$$

$$\frac{T}{1+\omega^2T^2} - \frac{\alpha T}{1+\alpha^2T^2\omega^2} = \frac{(T-\alpha T)+\alpha^2T^3\omega^2-\alpha T^3\omega^2}{(1+\omega^2T^2)(1+\alpha^2\omega^2T^2)}$$

$$(T-\alpha T)+\alpha^2T^3\omega^2-\alpha T^3\omega^2=0$$

$$\alpha T^2(-T+\alpha T)\omega^2+(T-\alpha T)=0 \longrightarrow \alpha T^2\omega^2-1=0, \quad \therefore \omega_{max}=\sqrt{\frac{1}{\alpha T^2}}=\frac{1}{T\sqrt{\alpha}}$$

$$\angle D(j\omega)\Big|_{\omega_{max}} = \frac{\angle\left(1+\frac{1}{\sqrt{\alpha}}j\right)}{\angle\left(1+\sqrt{\alpha}\,j\right)} = \tan^{-1}\left(\frac{1}{\sqrt{\alpha}}\right) - \tan^{-1}\left(\sqrt{\alpha}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{1+\alpha}}\right) - \sin^{-1}\left(\frac{\sqrt{\alpha}}{\sqrt{1+\alpha}}\right)$$

$$\sin(\phi_{max}) = \frac{1}{\sqrt{1+\alpha}}\frac{1}{\sqrt{1+\alpha}} - \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}}\frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} = \frac{1-\alpha}{1+\alpha}$$