자동제어 한방에 끝내기

6.Frequency Response

목차

Frequency domain

- Why?
- System에 진동하는 input을 넣으면?

System & Stability analysis

- Bode plot
- Stability margin
- Nyquist plot

Dynamic compensation

- Lead compensation
- Lag compensation

Frequency domain

- 이제는 '주파수'를 다룬다.
  - 주파수가 왜 중요하죠?

$$\sin(\omega t)$$

$$\frac{\omega}{s^2+\omega^2}$$

- EE history..
  - 제어공학은 왜 등장했는가?

Frequency domain

- 앞부분에서의 input은 step, ramp, …
  - 명시적으로 '진동수가 있는 input'에 대해 다룬 적이 없음

- Input이 진동하고 있을 때는 어떤 일이 일어나는가?
  - 공진…
  - BIBO stability

Frequency domain

- 진동하는 input
  - $\sin(\omega t)$
  - $\frac{\omega}{s^2+\omega^2}$
- Stable System을 다음과 같이 가정한다면 $(Re[p_i] < 0)$
  - $\frac{a_1}{s-p_1} + \frac{a_2}{s-p_2} + \frac{a_3}{s-p_3} + \cdots$

Frequency domain

$$Y(s) = U(s)G(s) = \left(\underbrace{\frac{\omega}{s^2+\omega^2}}_{} \overset{U(s)}{} \times \underbrace{\left[\frac{a_1}{s-p_1} + \frac{a_2}{s-p_2} + \frac{a_3}{s-p_3} + \cdots \right]}_{} \overset{G(s)}{} \right)$$

$$= \frac{k_1}{s-j\omega} + \frac{k_2}{s+j\omega} + \frac{b_1}{s-p_1} + \frac{b_2}{s-p_2} + \frac{b_3}{s-p_3} + \cdots$$

$$\rightarrow \mathcal{L}^{-1}\{Y(s)\} = k_1 e^{j\omega t} + k_2 e^{-j\omega t} + b_1 e^{p_1 t} + \cdots$$

$t \to \infty$

$|G(j\omega)| e^{j\angle G(j\omega)}$ $\quad |G(j\omega)| e^{-j\angle G(j\omega)}$

$G(j\omega)\frac{e^{j\omega t}}{2j} - G(-j\omega)\frac{e^{-j\omega t}}{2j}$

$|G(j\omega)| \left\{ \frac{-e^{-j(\omega t+\phi)} + e^{j(\omega t+\phi)}}{2j} \right\}$

$|G(j\omega)| \sin(\omega t+\phi)$

$k_1, k_2 = ?$

$$k_1 = \frac{\omega}{(s+j\omega)} \times G(s) \Big|_{s=j\omega} = \frac{\omega}{2j\omega} \cdot G(j\omega) = \frac{G(j\omega)}{2j}$$

$$k_2 = \frac{\omega}{s-j\omega} \times G(s) \Big|_{s=-j\omega} = -\frac{\omega}{2j\omega} \times G(-j\omega) = -\frac{G(-j\omega)}{2j}$$

Frequency domain

- 결국 input $U(t) = A\sin(\omega t)$ 시스템의 TF가 $G(s)$ 라면

- $Y(t \to \infty) = A|G(j\omega)| \times \sin\left(\omega t + \angle G(j\omega)\right)$

$2 + 2\sqrt{3}j \longrightarrow 4 \cdot \angle 60^\circ$

- $t \to \infty$일 때는 $G(j\omega) = \boldsymbol{|G(j\omega)|} e^{j\boldsymbol{\angle G(j\omega)}}$만 상관이 있다!

Bode plot

- $t \to \infty$일 때는 $G(j\omega) = \underbrace{\overset{\text{Magnitude}}{|\boldsymbol{G(j\omega)}|}}e^{j\overset{\text{phase}}{\angle \boldsymbol{G(j\omega)}}}$만 상관이 있다!

- 남은 변수는 ω뿐…

- ω에 따라 $|G(j\omega)|$와 $\angle G(j\omega)$를 그리자!

Bode plot-dB

$|1+\sqrt{3}j| \Rightarrow 2 \Rightarrow 0.3010 \Rightarrow 6\,\mathrm{dB}$

$\angle(1+\sqrt{3}j) \Rightarrow 60^\circ$

- ω에 따라 $|G(j\omega)|$와 $\angle G(j\omega)$를 그리자!
  - 그런데.. ω는 너무 넓은데..? -> log 스케일로 그린다.
  - 그럼 크기도 로그로 그려야 하는거 아닌가? -> 맞다.

$1 \rightarrow 10 \Rightarrow 20\,\mathrm{dB}$

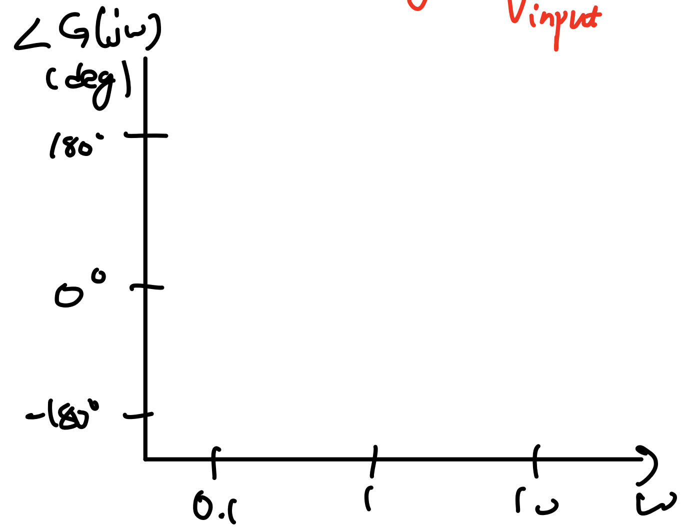
$1 \rightarrow 0.1 \Rightarrow -20\,\mathrm{dB}$

$1 \rightarrow 1 \Rightarrow 0\,\mathrm{dB}$

$\mathrm{dB} = 20 \cdot \log_{10} \frac{V_{output}}{V_{input}}$

Magnitude $|G(j\omega)|$

phase



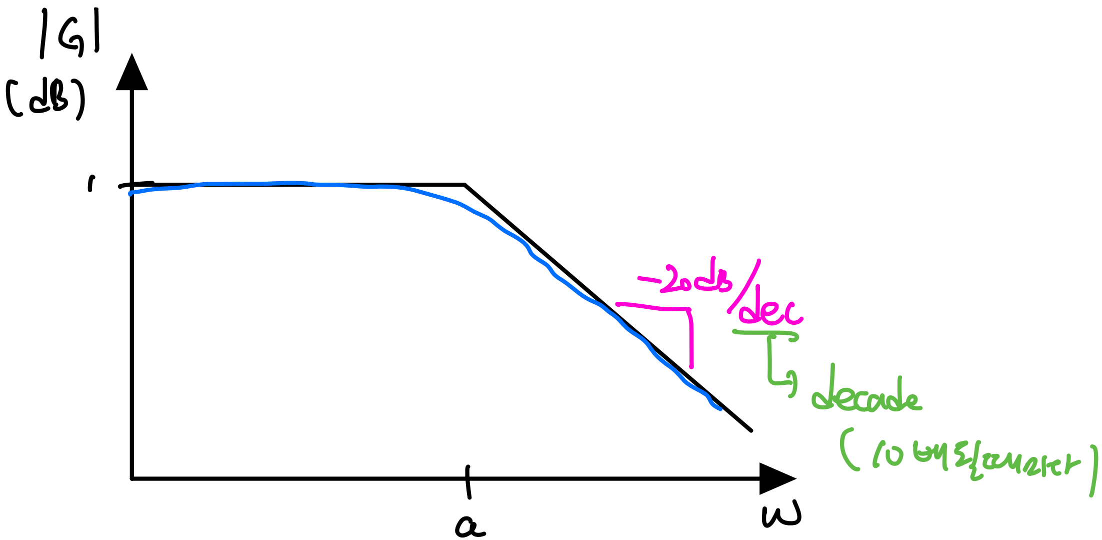
Bode plot

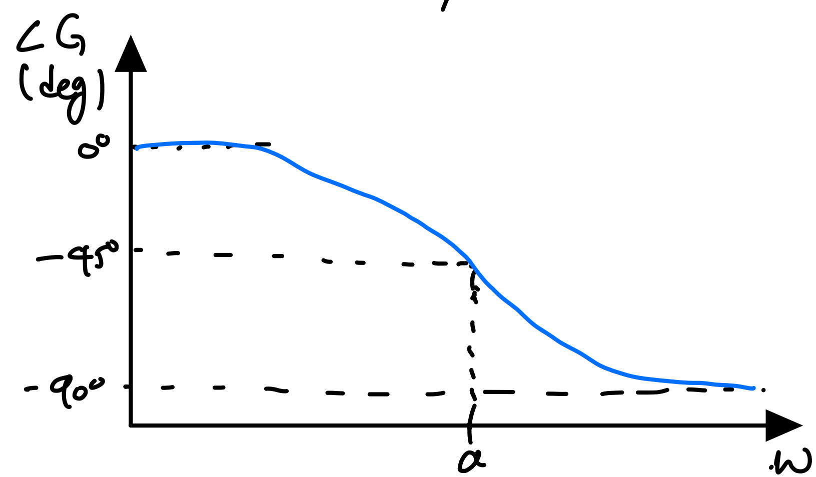
- Bode plot of $G(s) = \dfrac{1}{\frac{s}{a}+1}$

$$G(j\omega) = \frac{1}{j\left(\frac{\omega}{a}\right)+1}$$

$$\begin{cases} (\omega \ll a) & |G| \approx 1, \ \angle G \approx 0^\circ \\ (\omega \sim a) & |G| < 1, \ \angle G < 0^\circ \end{cases}$$

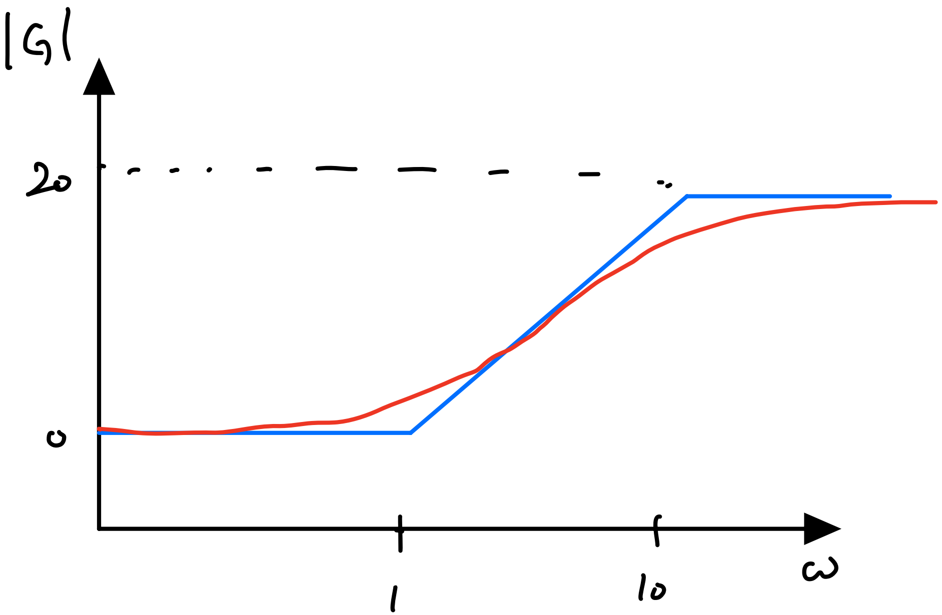
$(\omega \gg a)$ $|G| \ll 1$, $\angle G \approx -90^\circ$

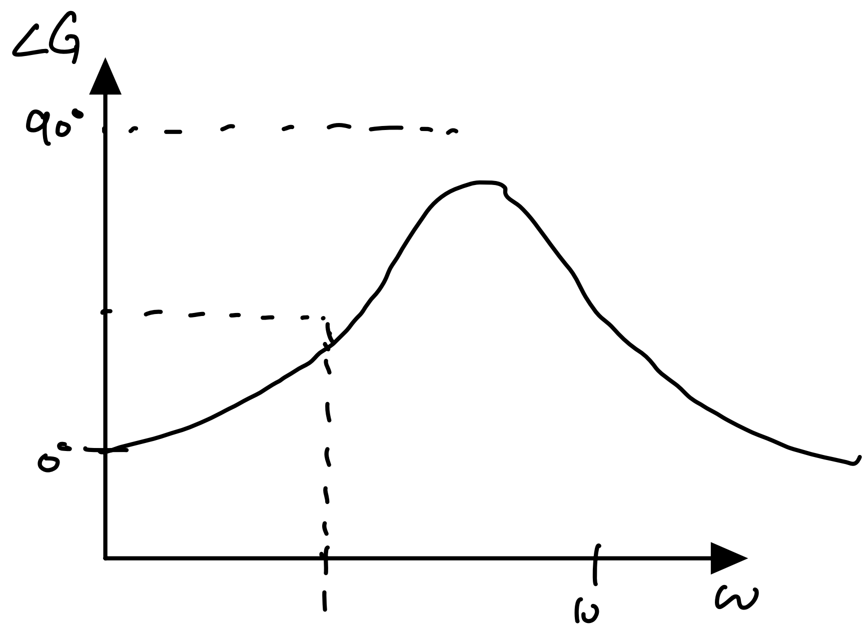




Bode plot

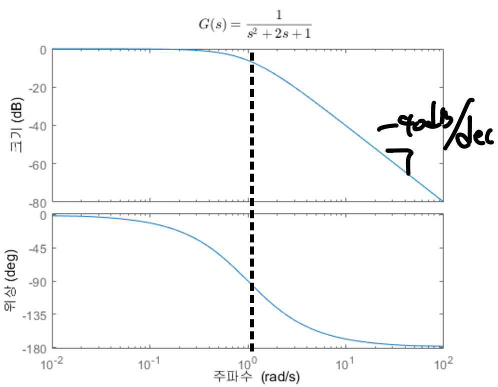
- Bode plot of $G(s) = \frac{\overset{\text{zero}=-1}{s+1}}{\underset{\text{pole: }-10}{0.1s+1}}$



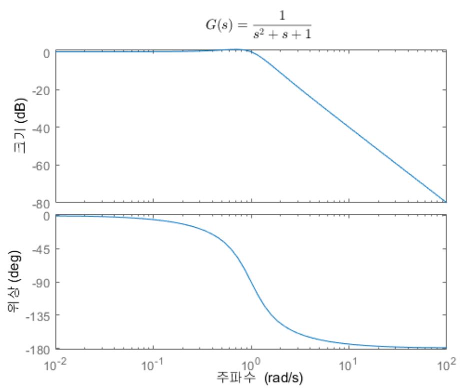


Bode plot

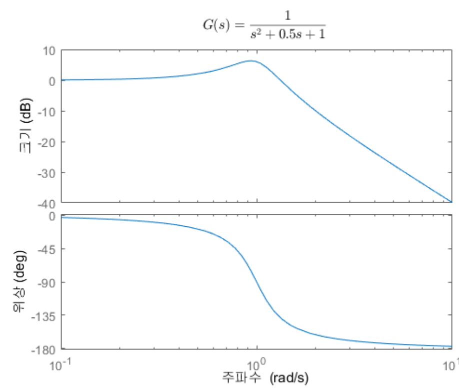
- Bode plot of $G(s) = \dfrac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$



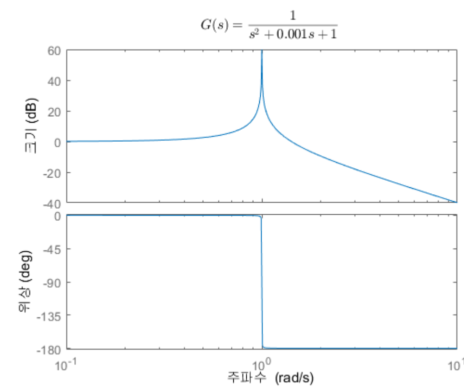
$\zeta = 1$



$\zeta = 0.5$



$\zeta = 0.25$



$\zeta = 0.0005$

Bode plot

- Matlab command

```
s = tf('s');
k = 1/(s^2+2*s+1);
bode(k)
title('$$G(s) = \frac{1}{s^2+2s+1}$$','Interpreter','latex')
```