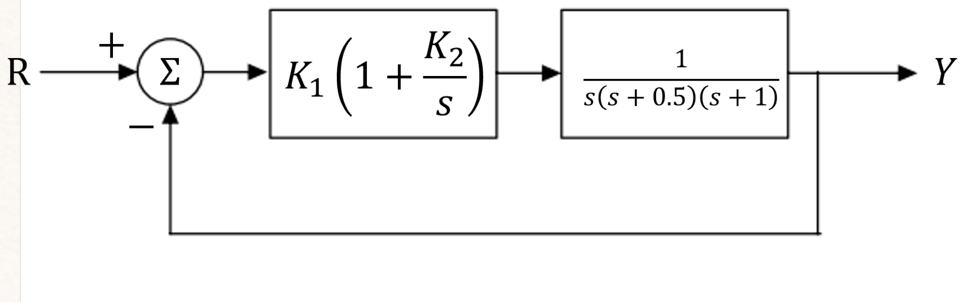
- Use Routh's criterion to determine the regions of (K_1, K_2) plane for which the system is stable



$$\text{CLTF} \longrightarrow \frac{K_1\left(1+\frac{K_2}{s}\right)\frac{1}{s(s+0.5)(s+1)}}{1+K_1\left(1+\frac{K_2}{s}\right)\frac{1}{s(s+0.5)(s+1)}}$$

$$\text{Char. eq} \longrightarrow s^2(s+0.5)(s+1) + K_1(s+K_2)$$

$$= s^4 + 1.5s^3 + 0.5s^2 + K_1 s + K_1K_2$$

Routh array $\longrightarrow$

$$s^4 \to 1 \qquad 0.5 \qquad K_1K_2$$

$$s^3 \to 1.5 \qquad K_1$$

$$s^2 \to \underbrace{-\frac{2}{3}\left(K_1-\frac{3}{4}\right)}_{A_1} \qquad K_1K_2$$

$$s^1 \to -\frac{1}{A_1}\left(1.5K_1K_2 - K_1A_1\right)$$

$$s^0 \to K_1K_2$$

Routh Criterion $\longrightarrow A_1 > 0, \ -\frac{1}{A_1}\left(1.5k_1k_2 - k_1A_1\right) > 0, \ k_1k_2 > 0$

1) $-\left(k_1 - \frac{3}{4}\right) > 0 \longrightarrow k_1 < \frac{3}{4}$

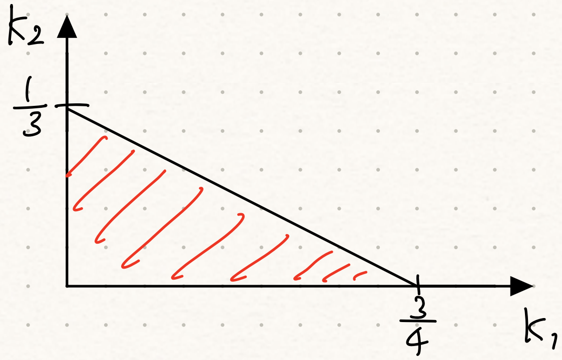
2) $k_1A_1 - \frac{3}{2}k_1k_2 > 0$, $\therefore -k_1 \times \frac{2}{3}\left(k_1 - \frac{3}{4}\right) - \frac{3}{2}k_1k_2 > 0$

$$-\frac{2}{3}k_1^2 + \frac{1}{2}k_1 - \frac{3}{2}k_1k_2 > 0$$

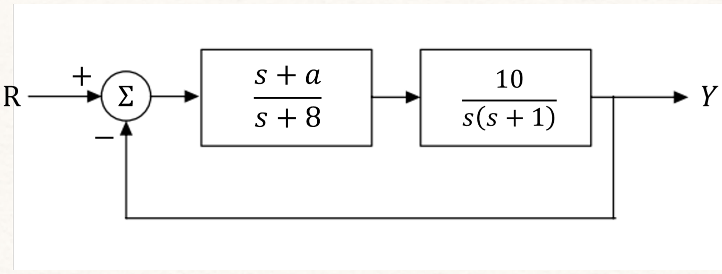
$$\frac{2}{3}k_1\left(k_1 - \frac{3}{4} + \frac{9}{4}k_2\right) < 0$$

$$\longrightarrow 0 < k_1 < \frac{3}{4} - \frac{9}{4}k_2 \quad \left(k_2 < \frac{1}{3}\right)$$

3) $k_1k_2 > 0$



- Draw the root locus of the given figure, with respect to a



$$CLTF = \frac{\frac{s+a}{s+8} \times \frac{10}{s(s+1)}}{1 + \frac{s+a}{s+8} \times \frac{10}{s(s+1)}}$$

$$s^2+9s+18$$

$$\frac{-9 \pm \sqrt{81-72}}{2}$$

$$\text{Char. eq} = (s+8)\,s\,(s+1) + (s+a)\times 10$$

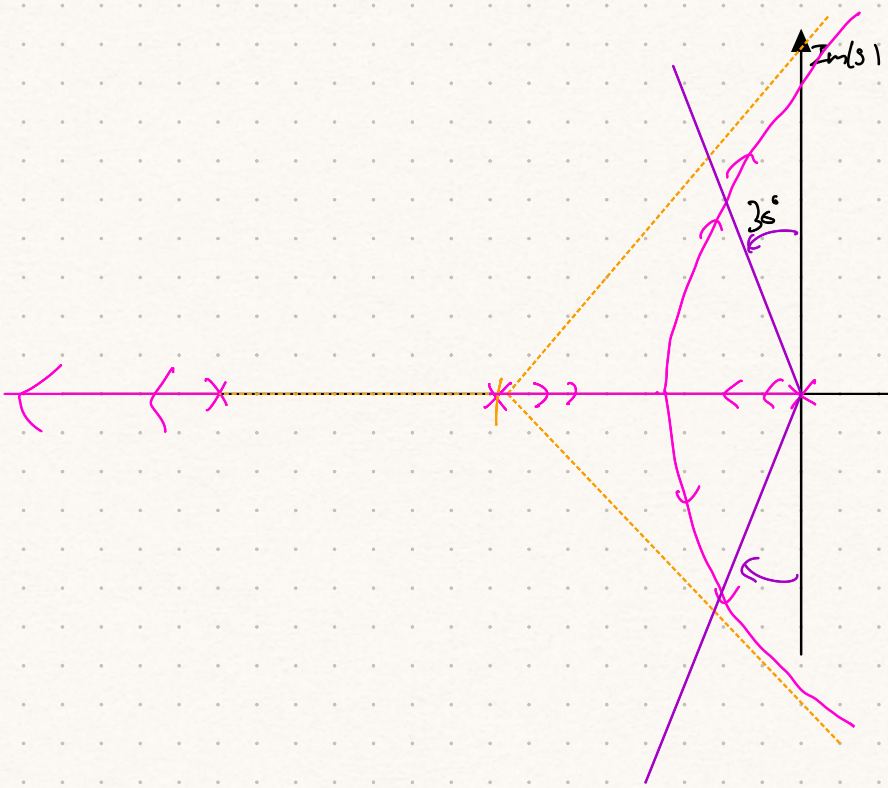
$$= s^3+9s^2+8s+10s+10a = s^3+9s^2+18s+10a$$

$$B(s) + KA(s) = 1 + K\frac{A(s)}{B(s)}$$

$$1 + a\,\frac{10}{s^3+9s^2+18s}$$

Zero: X

pole: $0, -3, -6$



$$\alpha = \frac{0-3-6}{3} = -3$$

$$\phi_l = \frac{180° + 360°(l-1)}{3}$$

$$= 60°, 180°, 300°$$

- Compute the value of a such that the damping ratio of the dominant CL poles are 0.5

$\phi = \sin^{-1}(\zeta)$

$t(-1+\sqrt{3}j) \longrightarrow s^3+9s^2+18s+10a$

$t\left(\frac{-1+\sqrt{3}j}{2}\right)$ $\qquad t^3+9t^2\bar{\omega}+18t\omega+10a=0$

$\omega \longrightarrow \omega^3-1=0$ $\qquad \longrightarrow 9t^2=18t \longrightarrow t=2$

$\omega^2=\bar{\omega}$

$8+36\left(\frac{-1-\sqrt{3}j}{2}\right)+36\left(\frac{-1+\sqrt{3}j}{2}\right)+10a=0$

$8-18-18+10a=0$, $\boxed{\therefore a=2.8}$