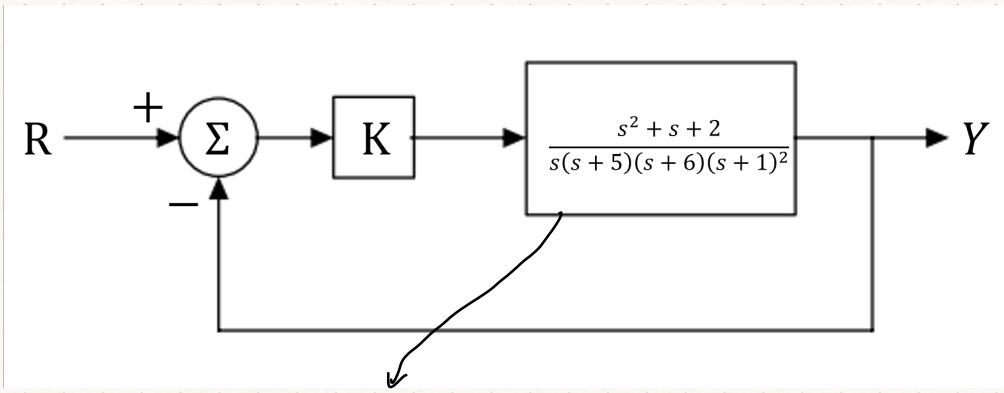
- Use Routh's criterion to find the range of K for which the system is stable, and use the root locus to confirm your calculations.



Char. eq = $1 + K\boxed{\quad} = s(s+5)(s+6)(s+1)^2 + K(s^2+s+2)$

$= s^5 + 13s^4 + 53s^3 + (71+K)s^2 + (30+K)s + 2K$

Routh Array : s^5 : 1 | 53 | 30+K

s^4 : 13 | 71+K | 2K

s^3 : $-\frac{(71+K) - 13\cdot 53}{13}$ = A_1 | $-\frac{2K - 13(30+K)}{13}$ = A_2

s^2 : $-\frac{13A_2 - (71+K)A_1}{A_1}$ = B | 2K

s^1 : $-\frac{2KA_1 - BA_2}{B}$ = C

s^0 : 2K

Stability analysis

$A_1 > 0, B > 0, C > 0, 2K > 0$

i) $-(71+K) - 13\cdot 53 > 0 \longrightarrow K < 618$

ii) $(71+K)A_1 - 13A_2 > 0$

$-(71+K)\frac{(71+K) - 13\cdot 53}{13} - 13\cdot\frac{2K - 13(30+K)}{13}$

$13^2 \times 30 + 13 \times 11K < -(71+K)(K-618)$

$\longrightarrow -80.156 < K < 484.156$

iii) $BA_2 - 2KA_1 > 0$

$-2K(K-618) < (13\cdot 30 + 11K)(K^2 - 404K - 38808)$

$\longrightarrow -80.156 < K < 478.234$ or $K > 484.156$

$0 < K < 478.234$

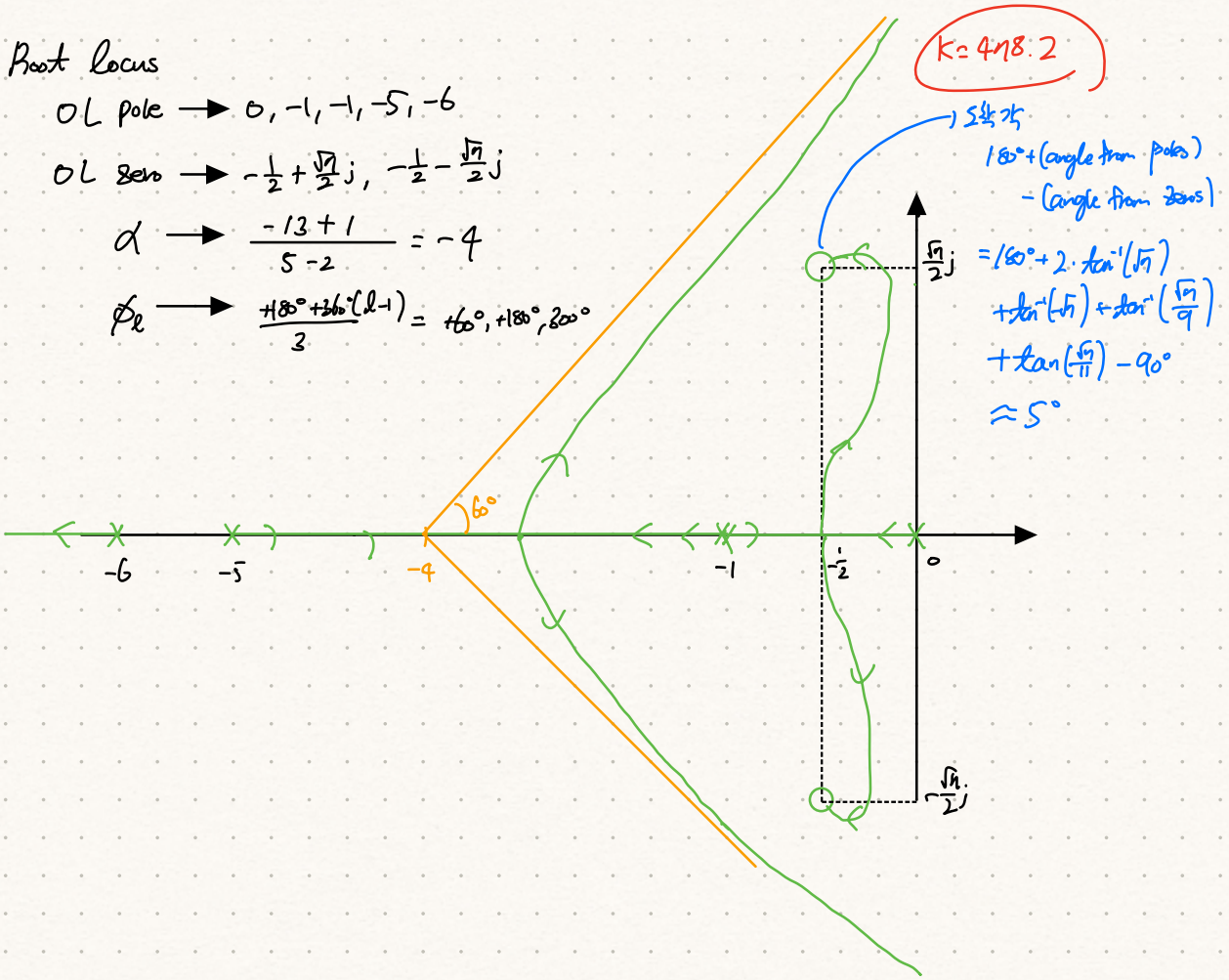
Root Locus

OL pole → $0, -1, -1, -5, -6$

OL zero → $-\frac{1}{2} + \frac{\sqrt{11}}{2}j,\ -\frac{1}{2} - \frac{\sqrt{11}}{2}j$

α → $\frac{-13+1}{5-2} = -4$

ϕ_l → $\frac{+180° + 360°(l-1)}{3} = +60°, +180°, 300°$

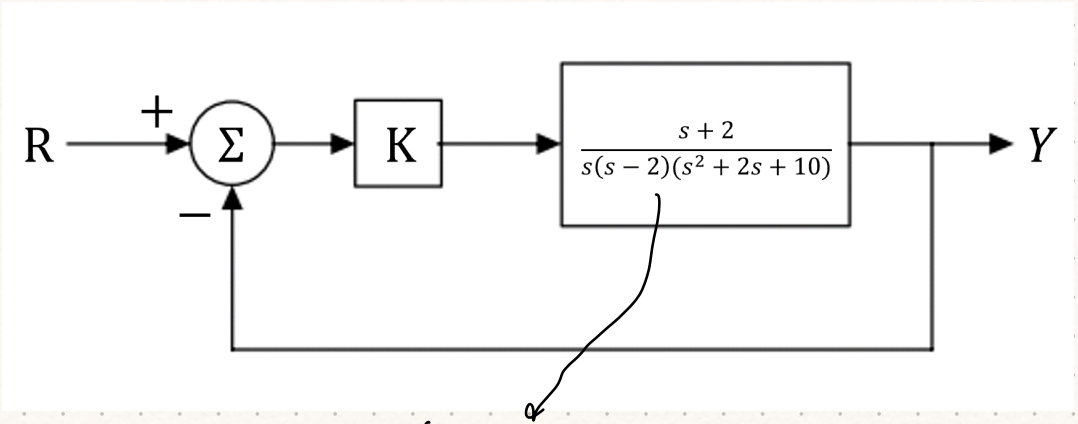


1 선택 가능

180° + (angle from poles) − (angle from zeros)

$= 180° + 2 \cdot \tan^{-1}(\sqrt{11}) + \tan^{-1}(-\sqrt{11}) + \tan^{-1}\left(\frac{\sqrt{11}}{9}\right) + \tan\left(\frac{\sqrt{11}}{11}\right) - 90°$

$\approx 5°$



CL Characteristic eq : $1+k(\quad) = s(s-2)(s^2+2s+10)+k(s+2)$

$= s^4+6s^2+(k-20)s+2k$

Routh Array

s^4	1	6	$2k$
s^3	0 (ε>0)	$k-20$	
s^2	$-\frac{1}{\varepsilon}(k-20-6\varepsilon)$ (A_1)	$2k$	
s^1	$-\frac{1}{A_1}\{2\varepsilon k - A_1(k-20)\}$		
s^0	$2k$		

$k-20-6\varepsilon<0$

$k<20-6\varepsilon$

$2\varepsilon k-A_1(k-20)<0$

$2\varepsilon k+\frac{1}{\varepsilon}(k-20-6\varepsilon)(k-20)>0$

$2\varepsilon^2 k+(k^2-40+6\varepsilon)k+400+120\varepsilon<0$

$k^2-(40+6\varepsilon-2\varepsilon^2)k+400+120\varepsilon<0$

불가능.

k 존재 X

pole : $0, -1\pm 2j, 2$

zero : -2

$\alpha = \frac{-2+2+2}{3} = \frac{2}{3}$

$\phi_l = 60°, 180°, 300°$