

자동제어 한방에 끝내기

5. Root Locus

목차

Root Locus(근궤적선)

- What? – 무엇인가?
- How? – 어떻게 쓰는가?

Root Locus 그리기

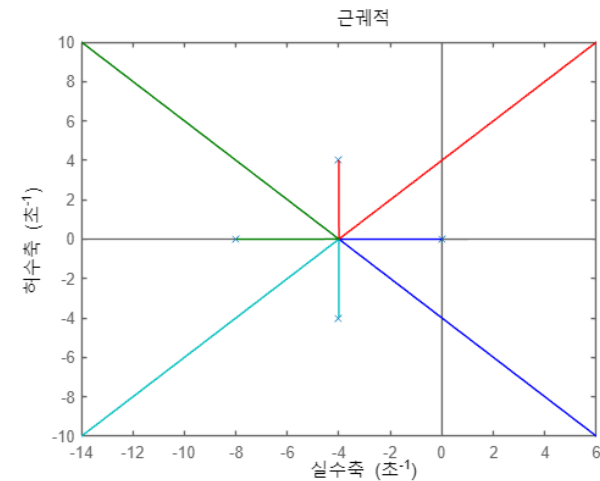
- Poles, Zeros
- Phase, Magnitude
- 출발각과 도착각

Root Locus – MATLAB?

```
s = tf('s')  
a = 1/(s*(s+8)*(s^2+8*s+32))  
rlocus(a)
```

s =
s
연속시간 전달 함수입니다.
a =
$$\frac{1}{s^4 + 16s^3 + 96s^2 + 256s}$$

연속시간 전달 함수입니다.



Root Locus – Examples

• $\frac{1}{s(s+8)\{(s+4)^2+16\}}$ zero: $\times$ pole: $0, -8, -4 \pm 4j$

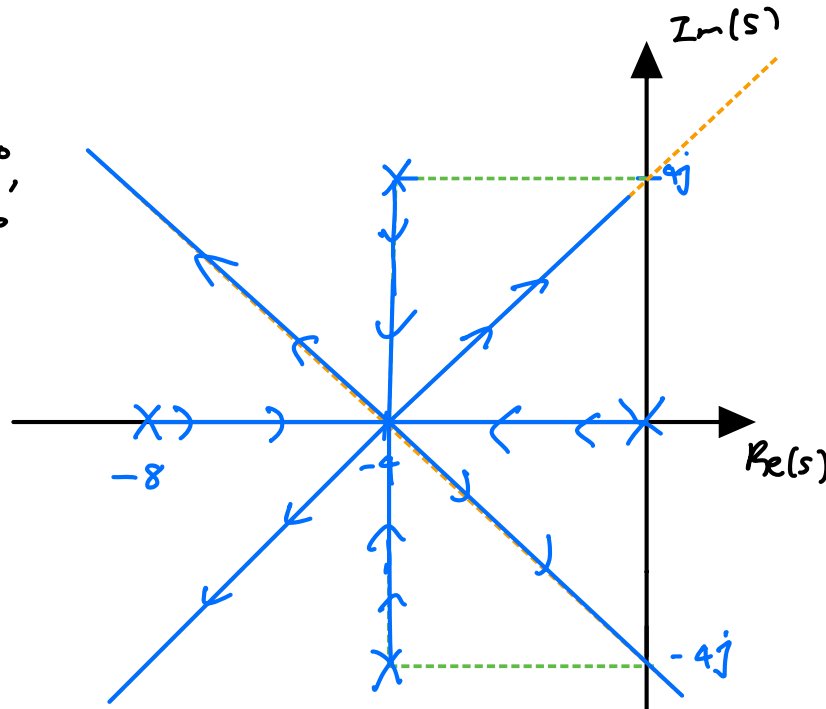
$$\sigma = \frac{\sum p - \sum z}{n - m} = \frac{-8 - 4 - 4}{4} = -4$$

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{4} = 45^\circ, 135^\circ, 215^\circ, 305^\circ$$

$$\sum \angle \psi_i - \sum \angle \phi_i = 180^\circ + 360^\circ(l-1)$$

$$\angle \phi_{(-4 \pm 4j)} = -135^\circ - 45^\circ - 90^\circ - 180^\circ$$

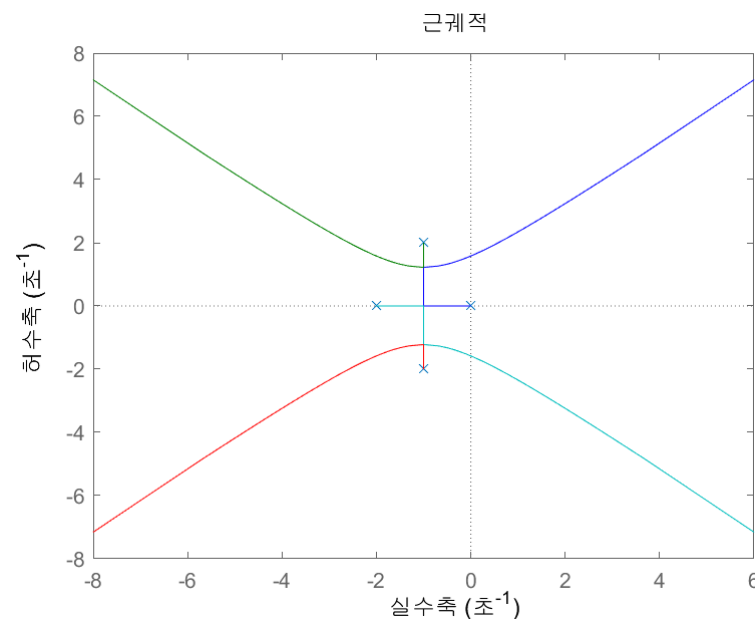
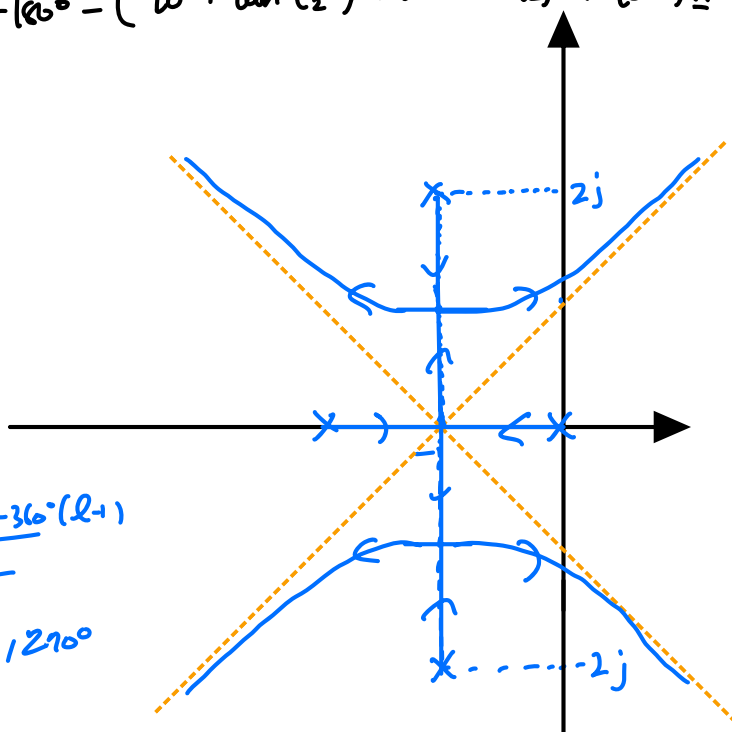
$$\theta_{(-4, 0j)} = \frac{180^\circ + 360^\circ(l-1)}{4} = 45^\circ, 135^\circ, 215^\circ, 305^\circ$$



Root Locus – Examples

$\bullet \frac{1}{s(s+2)\{(s+1)^2+4\}}$ pole = 0, -2, -1±2j, zero = 0, $\alpha = \frac{-4}{4} = -1$, $\phi_k = \frac{180^\circ + 360^\circ(k-1)}{4-0} = 45^\circ, 135^\circ, 215^\circ, 305^\circ$
 $\angle \phi_{(-1, 2j)} = -180^\circ - (90^\circ + \tan^{-1}(\frac{1}{2}) + 90^\circ - \tan^{-1}(\frac{1}{2}) + 90^\circ) = -90^\circ$

$$\theta_{(-1, 2j)} = \frac{180^\circ + 360^\circ(k-1)}{2} = 90^\circ, 270^\circ$$



Root Locus – Examples

- 분기하는 점의 s 값을 아는 방법
- $1 + KL(s) = 0$ 인 근이 모두 표시되어 있고, 보통 양쪽에서 와서 만난다.
 - K 의 값은 해당 위치에서 극대/극소
 - $(K)' = \left(-\frac{1}{L(s)}\right)'$ 가 해당 위치에서 0이 된다.
- 방금 전과 같은 그림에서 분기점의 값은?

```
syms t
t*(t+2)*((t+1)^2+4)

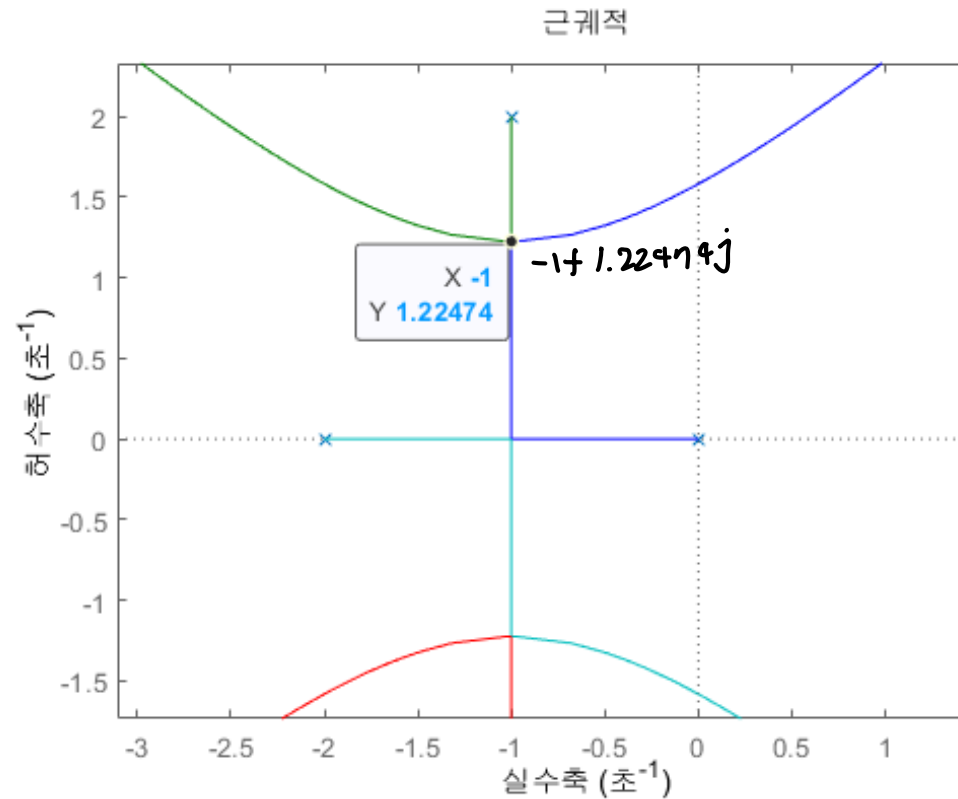
solve(simplify(diff(t*(t+2)*((t+1)^2+4)))==0,t)
|
```

```
ans = t ((t + 1)^2 + 4) (t + 2)
```

```
ans =
```

```
( -1
 -1 - 1.2247 i
 -1 + 1.2247 i )
```

Root Locus – Examples



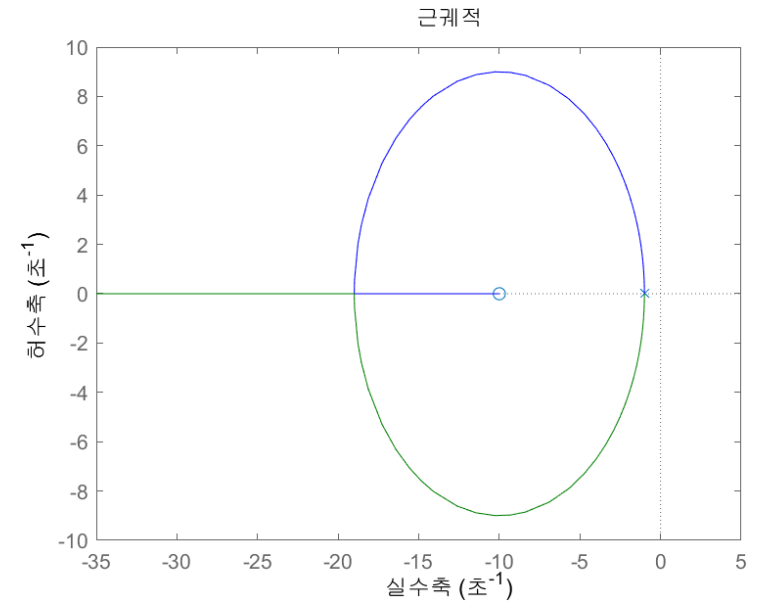
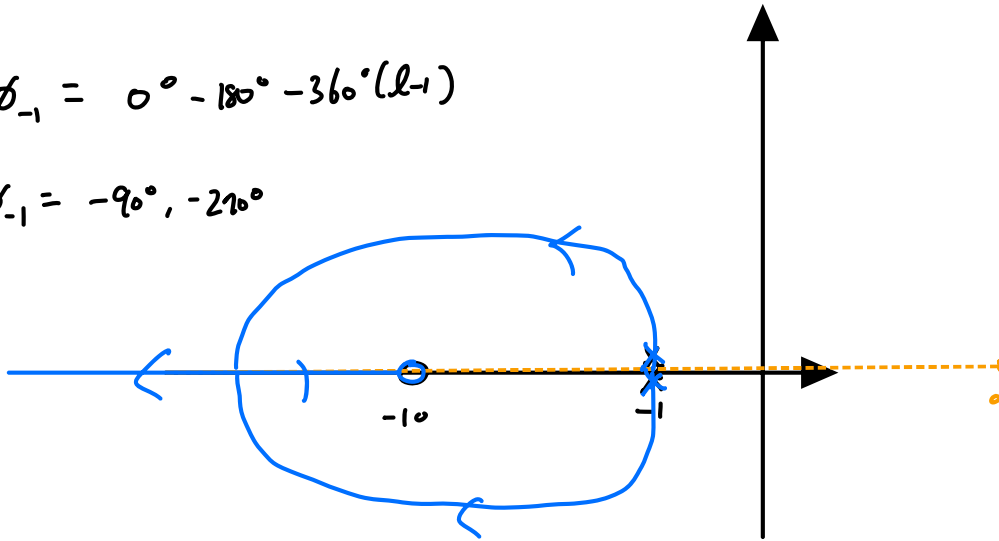
Root Locus – Examples

• $\frac{(s+10)}{(s+1)^2}$ poles: $-1, -1$ / zero: -10 , $\alpha = \frac{-1-1+10}{1} = 8$, $\phi_L = \frac{180^\circ + 360^\circ(l-1)}{1} = 180^\circ$

$$\sum \angle \psi_i - \sum \angle \phi_i = 180^\circ + 360^\circ(l-1)$$

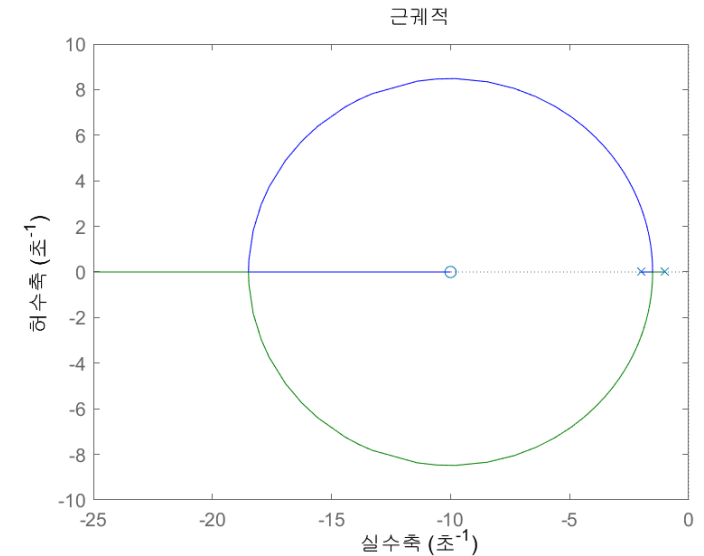
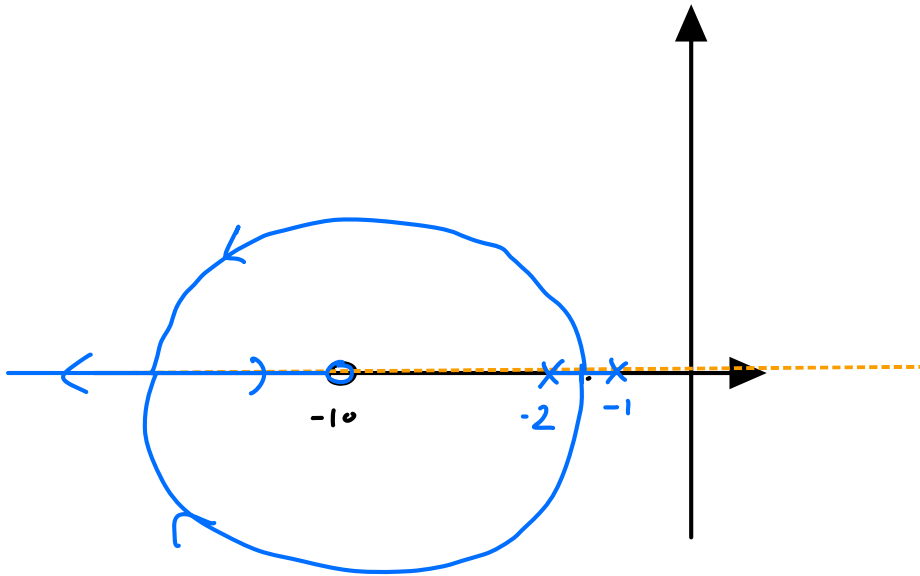
$$2\angle \phi_{-1} = 0^\circ - 180^\circ - 360^\circ(l-1)$$

$$\angle \phi_{-1} = -90^\circ, -270^\circ$$



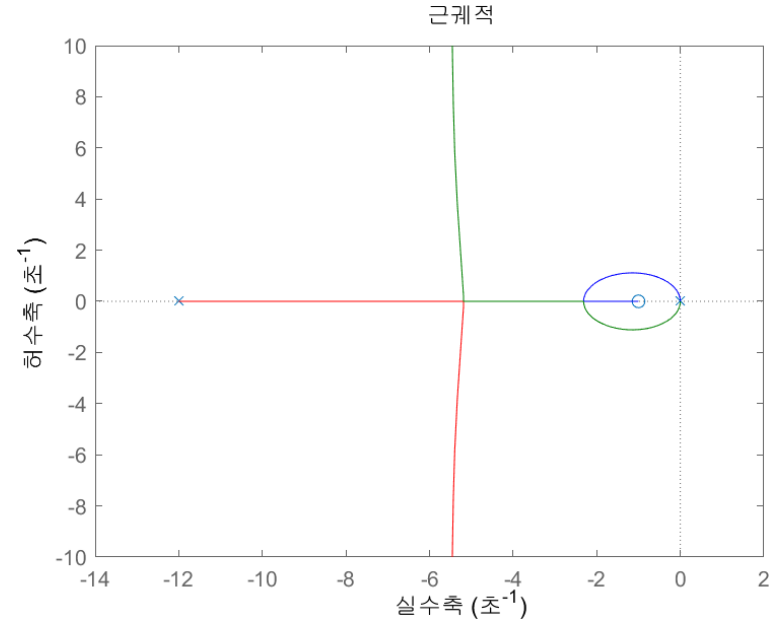
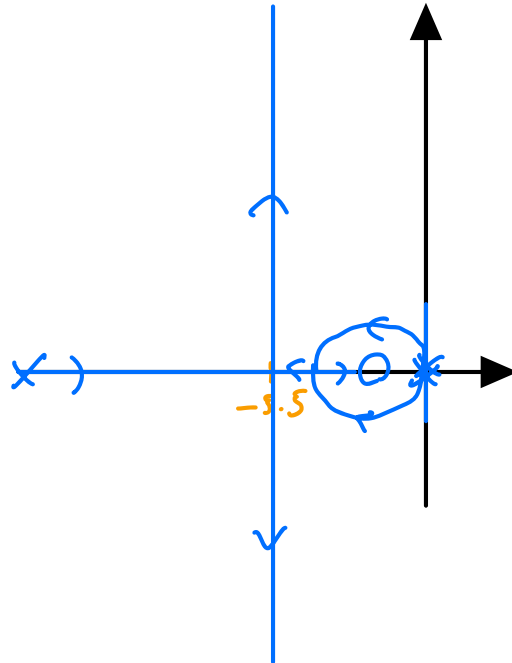
Root Locus – Examples

• $\frac{(s+10)}{(s+2)(s+1)}$ pole: $-2, -1$, zero: -10 $\alpha = \frac{-2-1+10}{1} = 7$, $\phi_2 = 180^\circ$



Root Locus – Examples

$\bullet \frac{(s+1)}{s^2(s+12)}$ pole: 0, 0, -12, Zero: -1 $\sigma = \frac{0+0-12+1}{2} = -5.5$, $\phi = \frac{180^\circ + 360^\circ(l-1)}{2} = 90^\circ, 270^\circ$

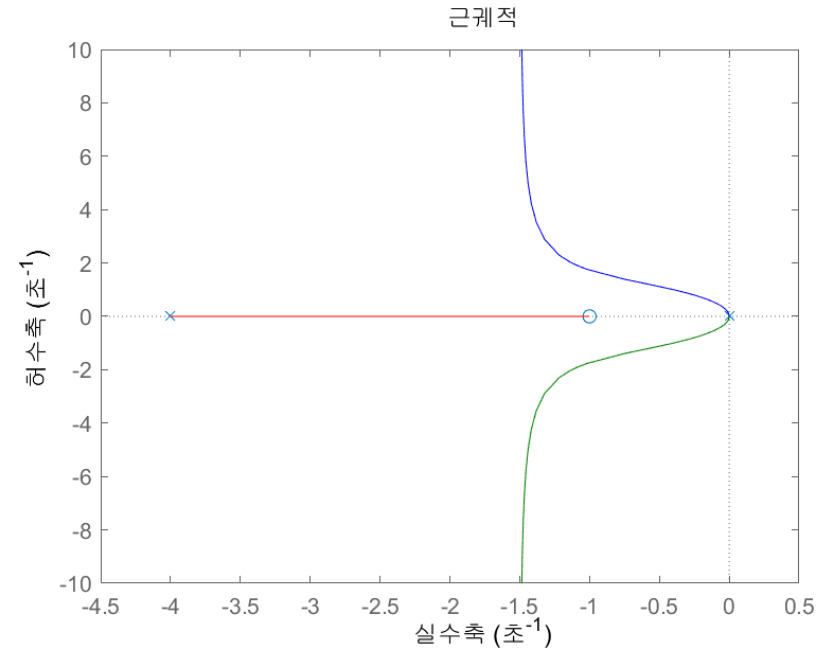
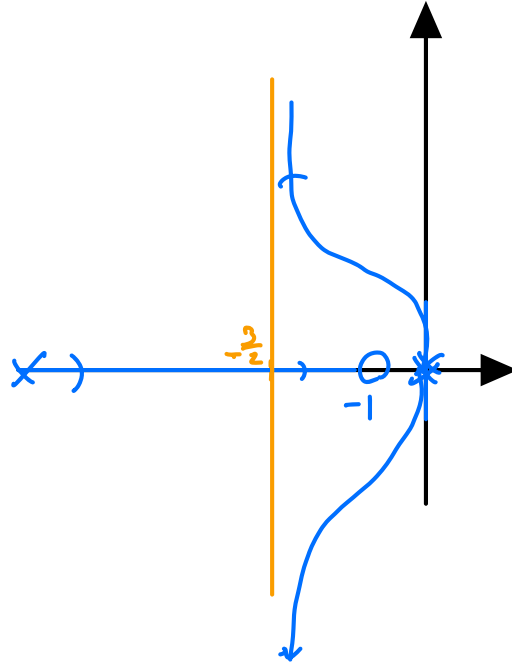


Root Locus – Examples

- $\frac{(s+1)}{s^2(s+4)}$

pole: 0, 0, -4 / zero: -1

$$\sigma = \frac{-4+1}{2} = -\frac{3}{2}, \quad \phi = 90^\circ, 270^\circ$$



Root Locus – Examples

- $\frac{(s+1)}{s^2(s+9)}$

$\text{pole} = 0, 0, -9, \text{Zero} = -1 \mid d = \frac{-9+1}{2} = -4, \phi_c = 90^\circ, 270^\circ$

