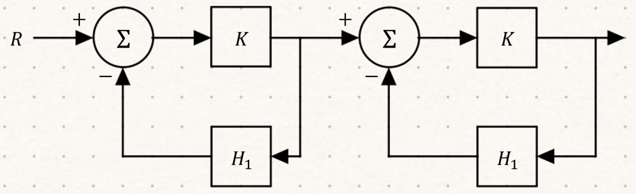
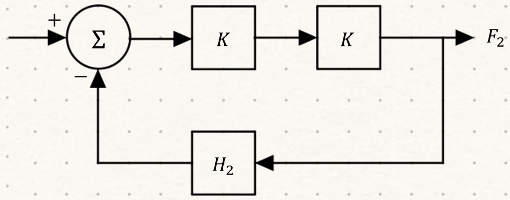
자동제어 한방에 끝내기

4. Feedback Controls

Select H_1 & H_2 so that $F_1 = F_2$, and discuss the sensitivity to changes in the overall gain due to changes in amplifier gain, $S_K^{F_1}$ & $S_K^{F_2}$





$$F_1 = \left(\frac{k}{1+kH_1}\right)\cdot\left(\frac{k}{1+kH_1}\right) R$$

$$F_2 = \frac{k^2}{1+k^2H_2}$$

$$(1+kH_1)^2 = 1+k^2H_2$$

$$k^2H_2 = 2kH_1 + k^2H_1^2$$

$$\therefore H_2 = H_1^2 + \frac{2H_1}{k}$$

$$S_k^{F_1} = \frac{\frac{\delta F_1}{F_1}}{\frac{\delta k}{k}} = \frac{k}{F_1}\cdot\frac{\partial F_1}{\partial k}$$

$$= \frac{k}{\frac{k^2}{(1+kH_1)^2}}\cdot\left\{\frac{2k}{(1+kH_1)^2} - \frac{k^2\cdot 2H_1}{(1+kH_1)^3}\right\}$$

$$= \frac{(1+kH_1)^2}{k}\cdot\left(\frac{2k+2k^2H_1-2k^2H_1}{(1+kH_1)^3}\right)$$

$$= \boxed{\frac{2}{(1+kH_1)}}$$

$$S_k^{F_2} = \frac{\frac{\delta F_2}{F_2}}{\frac{\delta k}{k}} = \frac{k}{F_2}\cdot\frac{\partial F_2}{\partial k}$$

$$= \frac{k}{\frac{k^2}{1+k^2H_2}}\cdot\left(\frac{2k}{1+k^2H_2} - \frac{k^2\cdot 2kH_2}{(1+k^2H_2)^2}\right)$$

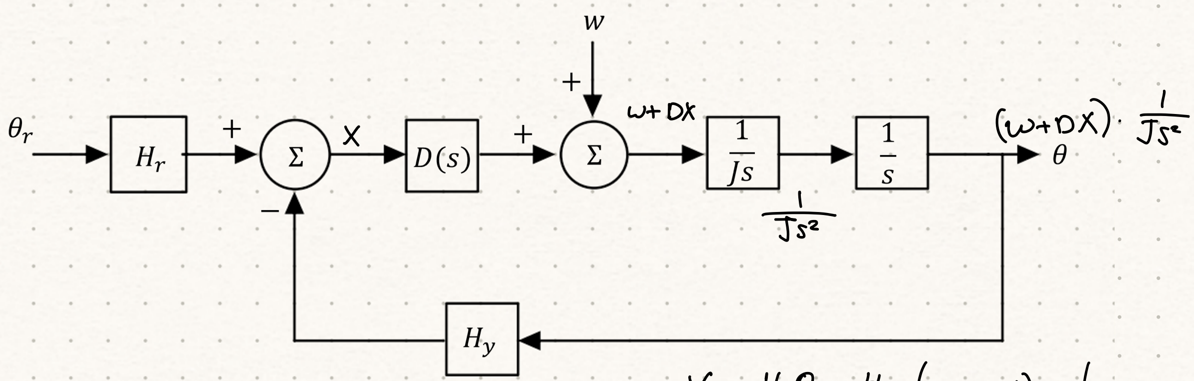
$$= \frac{1+k^2H_2}{k}\left(\frac{2k+2k^3H_2-2k^3H_2}{(1+k^2H_2)^2}\right) = \boxed{\frac{2}{1+k^2H_2}}$$

$$S_k^{F_1} = \frac{2}{1+kH_1}$$

$$S_k^{F_2} = \frac{2}{1+k^2H_2}$$

$$= \frac{2}{1+k^2H_1^2+2kH_1}$$

$$= \frac{2}{(1+kH_1)^2}$$



$$X = H_r\theta_r - H_y(w+DX)\cdot\frac{1}{Js^2}$$

Satellite attitude control problem

(a) Use PID control, determine the system type & error constant w.r.t. reference inputs
(b) Use PID control, determine the system type & error constant w.r.t. disturbance inputs

$$X = H_r\theta_r - \frac{H_y w}{Js^2} - \frac{H_y DX}{Js^2} \longrightarrow \left(1+\frac{H_y D}{Js^2}\right)X = H_r\theta_r - \frac{H_y}{Js^2}w$$

$$X = \frac{H_r\theta_r - \frac{H_y}{Js^2}w}{1+\frac{H_y D}{Js^2}} \longrightarrow \theta = \frac{(w+DX)}{Js^2} = \frac{w}{Js^2} + \frac{D}{Js^2}\,\frac{H_r\theta_r - \frac{H_y}{Js^2}w}{1+\frac{H_y D}{Js^2}}$$

$$= \frac{w}{Js^2} + \frac{DH_r\theta_r - \frac{H_y D}{Js^2}w}{Js^2+H_y D}$$

$$= \frac{DH_r\theta_r}{Js^2+H_y D} + \frac{\cancel{Js^2}w + \cancel{H_y D w} - \cancel{H_y D w}}{(\cancel{Js^2})(Js^2+H_y D)}$$

$$= \frac{DH_r}{Js^2+H_y D}\theta_r + \frac{1}{Js^2+H_y D}w$$

$R(s) - Y(s) = \left(1 - \frac{Y(s)}{R(s)}\right)\cdot R(s)$

$$\text{(a)}\quad E(s)\Big|_{w=0} = \lim_{s\to 0} s\cdot\left(1 - \frac{H_y D}{Js^2+H_y D}\right)\cdot\frac{1}{s^n} = \lim_{s\to 0}\frac{Js^2}{Js^2+H_y D}\cdot\frac{1}{s^{n-1}}$$

$D(s) = \text{PID Control} \longrightarrow D(s) = k_p + \frac{k_I}{s} + k_D s$

$= (k_p s + k_I + k_D s^2)\cdot\frac{1}{s}$

$$= \lim_{s\to 0}\frac{Js^3}{Js^3+(k_D s^2+k_p s+k_I)H_y}\,\frac{1}{s^{n-1}}$$

Step input $\longrightarrow$ $n=1$

(n-1)이 3일때 error가 상수로 발생 $\longrightarrow$ $\boxed{\text{Type 3}}$

$$\text{Error} = \frac{J}{H_y k_I} = \frac{1}{e_j}$$

error constant $= \frac{H_y k_I}{J}$

$\left(R(s) - Y(s)\right) = 0 - Y(s) = \frac{-Y(s)}{W(s)} \cdot W(s)$

$$(b)\quad E(s)\Big|_{\theta_r=0} = \lim_{s\to 0} s\cdot\left(-\frac{1}{Js^2+H_yD}\right)\cdot\frac{1}{s^n} = \lim_{s\to 0}\left(-\frac{s}{Js^3+(K_Ds^2+K_ps+K_I)H_y}\right)\frac{1}{s^{n-1}}$$

$(n-1)$이 1일 때 error가 상수로 일정 $\longrightarrow$ $\boxed{\text{Type 1}}$

$$\text{Error} = \frac{1}{H_y K_I} = \frac{1}{e_v}$$

$$\text{error constant} = H_y K_I$$