자동제어 한방에 끝내기

4. Feedback Controls

목차

- Closed loop systems
- Pole-zero cancellation
- Sensitivity
- System types
- PID controls

Sensitivity

- Input Signal이 변하는 정도에 따른 Output Signal의 변화

$$S_G^{TF} \equiv \frac{\frac{\delta TF}{TF}}{\frac{\delta G}{G}} = \frac{G}{TF}\frac{\delta TF}{\delta G} = \frac{G}{TF}\frac{\partial TF}{\partial G}$$

open-loop

$$\rightarrow D \rightarrow G \rightarrow Y \quad : TF = DG$$

$$\frac{\frac{\cancel{D}\delta G}{\cancel{D}G}}{\frac{\delta G}{G}} = 1 \quad \Big/ \quad \frac{\cancel{G}}{D\cancel{G}} \cdot \frac{\partial DG}{\partial G} = 1$$

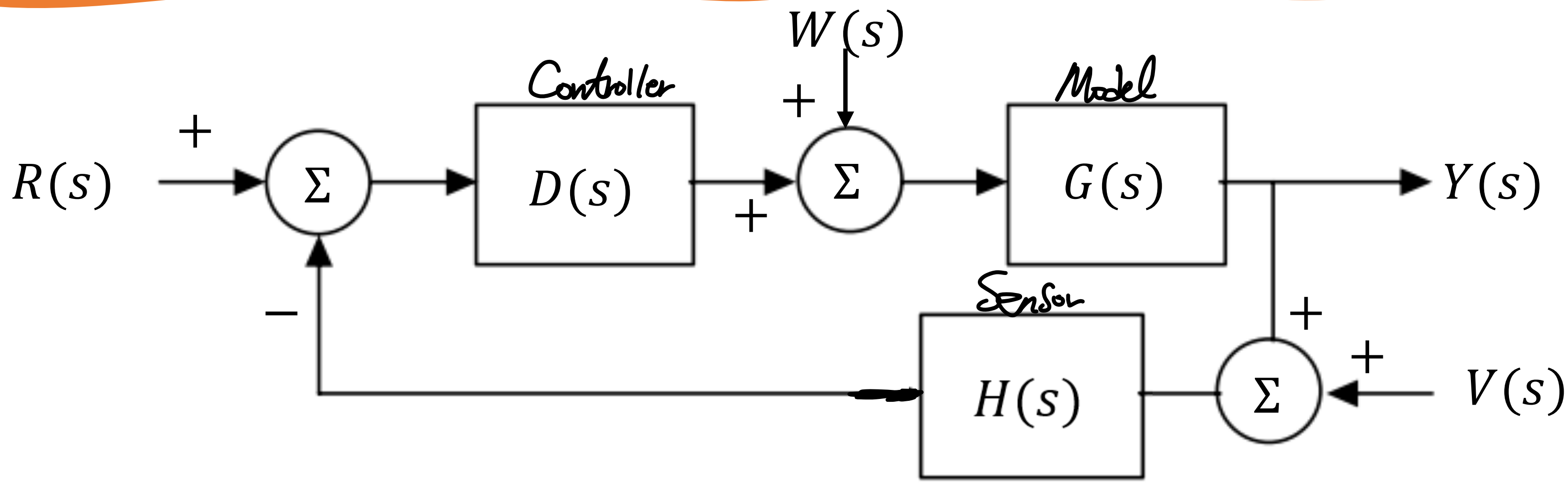
D

closed-loop : $TF = \frac{DG}{1+DG}$

$$\frac{\frac{\frac{D\delta G}{1+D\delta G}}{\frac{DG}{1+DG}}}{\frac{\delta G}{G}} \quad \Big/ \quad \frac{G}{\frac{DG}{1+DG}} \cdot \frac{\partial\left(\frac{DG}{1+DG}\right)}{\partial G} = \frac{1+DG}{D}\left\{\frac{D}{1+DG} - \frac{D^2G}{(1+DG)^2}\right\}$$

$$= 1 - \frac{DG}{1+DG} = \frac{1}{1+GD}$$

Sensitivity



- $Y(s) = \frac{GD}{1+GDH}R + \frac{G}{1+GDH}W - \frac{GDH}{1+GDH}V$

Sensitivity

- $Y(s) = \frac{GD}{1+GDH}R + \frac{G}{1+GDH}W - \frac{GDH}{1+GDH}V$

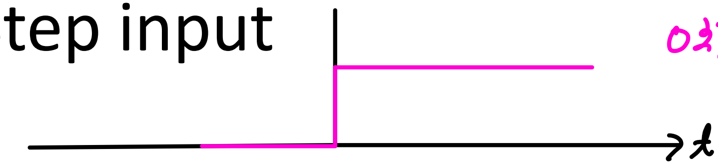
- S_D^{TF} $\quad \frac{\frac{\delta TF}{TF}}{\frac{\delta D}{D}} = \frac{D}{TF} \cdot \frac{\partial TF}{\partial D} = \frac{D}{\frac{GD}{1+GDH}} \cdot \left\{ \frac{G}{1+GDH} - \frac{GD \cdot GH}{(1+GDH)^2} \right\}$

$$= \frac{1+GDH}{G} \left\{ \frac{G}{(1+GDH)^2} \right\} = \frac{1}{1+GDH}$$

- S_H^{TF} $\quad \frac{\frac{\delta TF}{TF}}{\frac{\delta H}{H}} = \frac{H}{TF} \cdot \frac{\partial TF}{\partial H} = \frac{H}{\frac{GD}{1+GDH}} \cdot \left\{ - \frac{GD \cdot GD}{(1+GDH)^2} \right\} = - \frac{GDH}{1+GDH}$

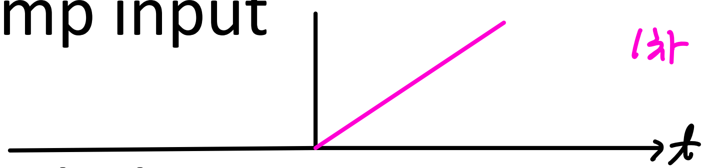
System types

- 몇 차의 input까지 tracking할 수 있는가?
  - Step input



0차 input $R(t)=k \xrightarrow{\mathcal{L}} R(s)=\frac{k}{s}$

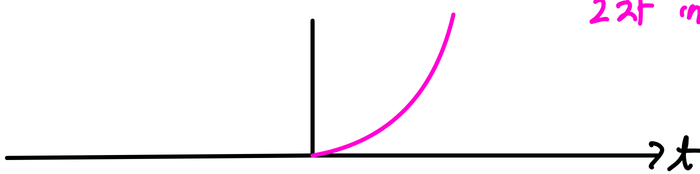
  - Ramp input



1차 input $R(t)=kt \xrightarrow{\mathcal{L}} R(s)=\frac{k}{s^2}$

  - Parabolic input

2차 input $R(t)=kt^2 \xrightarrow{\mathcal{L}} R(s)=\frac{k}{s^3}$



System types & Constant

- 몇 차의 input까지 tracking할 수 있는가?

$$E = \frac{1}{1+GD_{cl}}R, \therefore e_{ss} = \lim_{s\to 0} sE(s) = s\frac{1}{1+GD_{cl}}R(s)$$

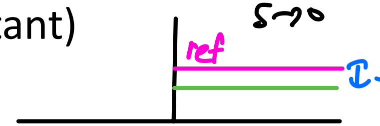
$R(s) = \frac{1}{s}$, $GD_{cl} = \frac{1}{s+a}$

$r = \frac{GD_{cl}}{1+GD_{cl}}$ $\quad E = R - r$

- GD_{cl}이 원점에 pole을 갖지 않을 때 -> Type 0 System

$\frac{1}{1+K_p}$(Position constant)

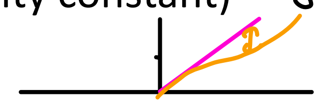
$$\lim_{s\to 0} \frac{1}{1+\frac{1}{s+a}} = \frac{1}{1+\frac{1}{a}}$$



- GD_{cl}이 원점에 pole을 1개 가질 때 -> Type 1 System

$GD_{cl} = \frac{1}{s(s+a)}$

$\frac{1}{K_v}$(Velocity constant)



$$s\frac{1}{1+\frac{1}{s(s+a)}}\cdot R(s) = \frac{\cancel{s^2}\,1\,(s+a)}{s(s+a)+1}\cdot R(s), \quad R(s) = \frac{1}{\cancel{s^2}}$$

- GD_{cl}이 원점에 pole을 2개 가질 때 -> Type 2 System

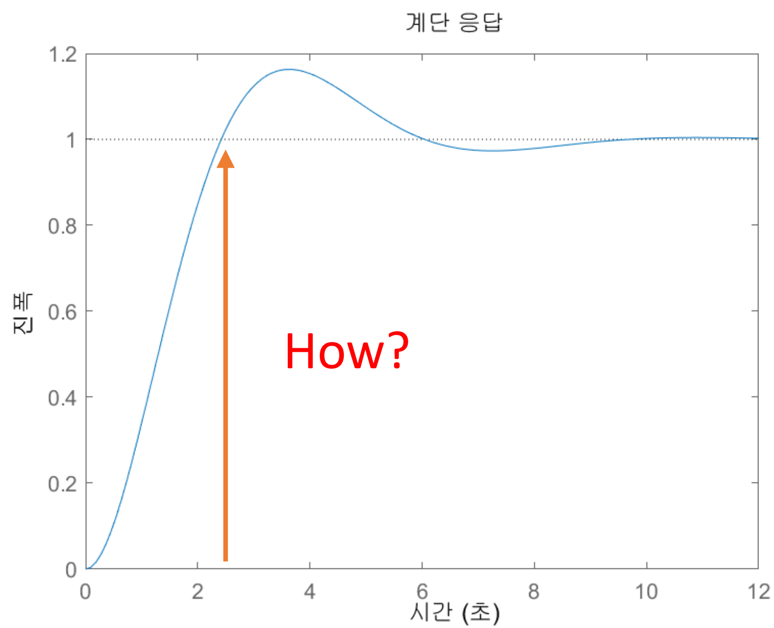
$GD_{cl} = \frac{b}{s^2(s+a)}$

$\frac{1}{K_a}$(Acceleration constant)

$$s\frac{1}{1+\frac{b}{s^2(s+a)}}\cdot R(s) = \frac{s^3(s+a)}{s^2(s+a)+b} \quad \frac{1}{s^2}$$

PID Controls - Introduction

- 지금까지 논의한 것은 system의 구성 – 얼마나 stable한가?
- 지금부터는 Controller $D(s)$를 어떻게 만들 것인가에 대해 논의



PID Controls - Introduction

- 이런 block diagram에서는 어떻게 나타낼 것인가?