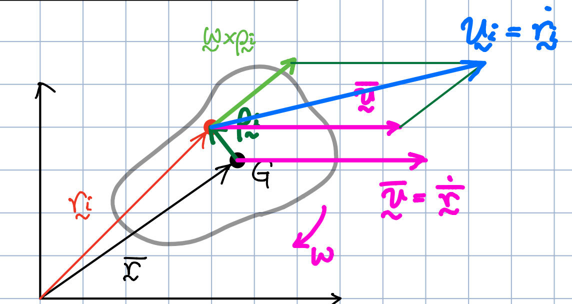
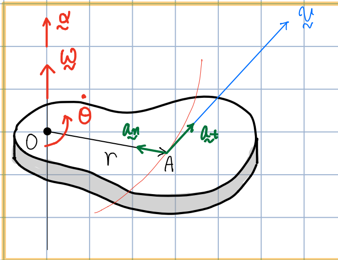
1. Linear Momentum





$v = \omega \times r$

$a_n = \omega \times (\omega \times r)$

$a_t = \alpha \times r$

$a = a_n + a_t$

$$\boxed{G = \sum m_i v_i} = \sum m_i \dot{r}_i$$

$$= d(\sum m_i r_i)/dt \leftarrow$$

$$= d(m\bar{r})/dt$$

$$= m\dot{\bar{r}}$$

$$\sum m_i r_i = (\sum m_i)\bar{r}$$

$$\sum m_i (r_i - \bar{r}) = 0$$

$$= \rho_i$$

$$\sum m_i \rho_i = 0$$

$$\boxed{G = \sum m_i v_i} \leftarrow v_i = \bar{v} + \omega \times \rho_i$$

$$= \sum m_i (\bar{v} + \omega \times \rho_i)$$

$$= \sum m_i \bar{v} + \omega \times \sum m_i \rho_i \leftarrow \sum m_i \rho_i = 0$$

$$= \sum m_i \bar{v}$$

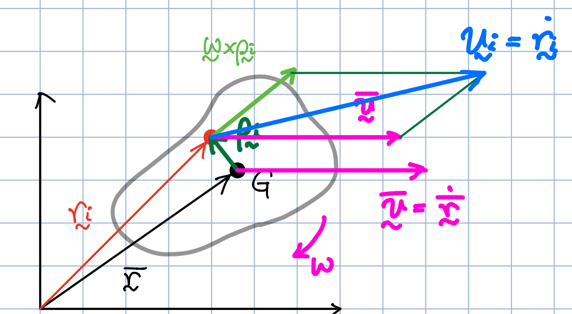
$$\boxed{G = m\bar{v}}$$

$$\sum F = \dot{G} \leftarrow \dot{G} = dG/dt$$

$$\int_{t_1}^{t_2} \sum F\, dt = \int_1^2 dG = G_2 - G_1$$

$$\boxed{\therefore G_1 + \int_{t_1}^{t_2} \sum F\, dt = G_2}$$

2. Angular Momentum



$$H_G = \sum \rho_i \times m_i \dot{\rho}_i \leftarrow \dot{\rho}_i = \omega \times \rho_i$$

$$= \sum \rho_i \times m_i (\omega \times \rho_i)$$

$$H_G = \sum \rho_i^2 m_i \omega \leftarrow I = \sum m_i \rho_i^2$$

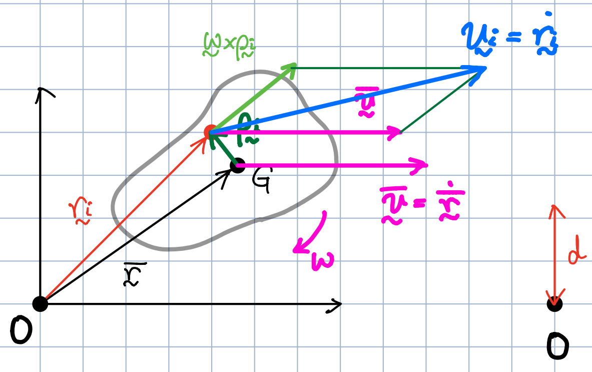
$$= I\omega$$

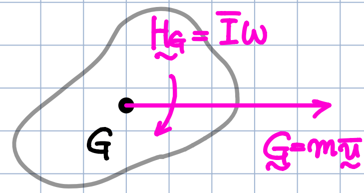
$$\dot{H}_G = \cancel{\sum \dot{\rho}_i \times m_i \dot{\rho}_i} + \sum \rho_i \times m_i \ddot{\rho}_i$$

$$= \sum M_G$$

$$\int_1^2 dH_G = \int_{t_1}^{t_2} \sum M_G\, dt$$

$$\boxed{\therefore (H_G)_1 + \int_{t_1}^{t_2} \sum M_G\, dt = (H_G)_2}$$

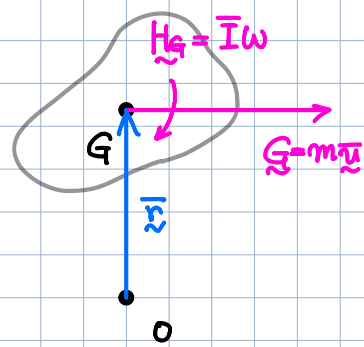




$$\boxed{H_O = \bar{I}\omega + m\bar{v}d} \leftarrow \begin{array}{l} \bar{v} = \bar{r}\omega \\ d = \bar{r} \end{array}$$

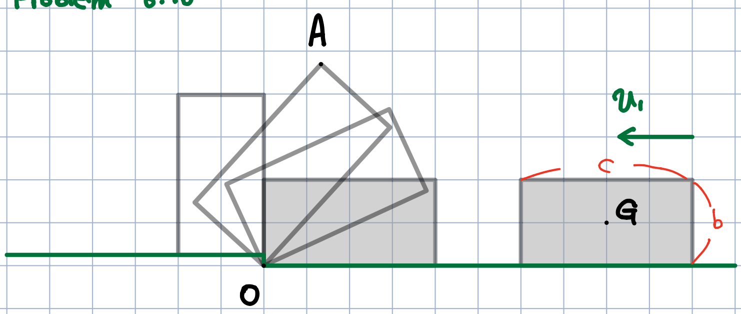
$$= \bar{I}\omega + m\bar{r}^2\omega$$

$$= (\bar{I} + m\bar{r}^2)\omega \leftarrow I_O = \bar{I} + m\bar{r}^2$$



$$\boxed{H_O = I_O\omega}$$

Problem > 6.16

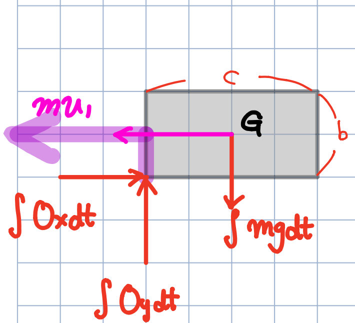


a) Compute u_1 when reaching the standing position

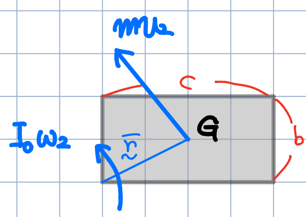
b) Compute the percent energy loss n for $b=c$

a) u_1

1. Collision



2. The subsequent Rotation



$\boxed{(H_o)_1 = (H_o)_2} \longrightarrow \frac{b}{2}\cdot \cancel{m}u_1 = \frac{\cancel{m}}{3}(b^2+c^2)\cdot\omega_2 \qquad \therefore\ \omega_2 = \frac{3bu_1}{2(b^2+c^2)}$

$\boxed{(H_o)_1 = \frac{b}{2}\cdot mu_1} \longleftarrow H_G = \Sigma \rho_i \times m\dot{\rho}_i$

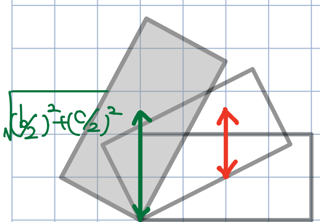
$\boxed{(H_o)_2 = I_o\omega_2} \longleftarrow I_o = I_G + m\bar{r}^2 \longleftarrow I_G = \frac{1}{12}m(b^2+c^2)\ ,\ \bar{r}^2 = \left(\frac{b}{2}\right)^2+\left(\frac{c}{2}\right)^2$

$= \frac{1}{12}m(b^2+c^2) + m\left\{\left(\frac{b}{2}\right)^2+\left(\frac{c}{2}\right)^2\right\}$

$= \frac{1}{12}m(b^2+c^2) + \frac{1}{4}m(b^2+c^2)$

$= m(b^2+c^2)\cdot\left(\frac{1}{12}+\frac{1}{4}\right) = \frac{m}{3}(b^2+c^2)$

3. Rotation



$\boxed{T_2+V_2 = T_3+V_3}$

$\frac{1}{2}I_o\omega_2^2 + 0 = 0 + mg\left\{\sqrt{(b/2)^2+(c/2)^2} - b/2\right\}$

$\cancel{\frac{1}{2}}\cdot\frac{\cancel{m}}{\cancel{3}}\cancel{(b^2+c^2)}\ \frac{\overset{3}{\cancel{9}}b^2u_1^2}{4(b^2+c^2)^{\cancel{2}}} = \frac{\cancel{m}g}{\cancel{2}}\left\{\sqrt{b^2+c^2} - b\right\}$

$u_1^2 = \frac{4}{3}\cdot\frac{g}{b^2}(b^2+c^2)\left\{\sqrt{b^2+c^2}-b\right\}$

$u_1 = 2\cdot\sqrt{\frac{g}{3b^2}(b^2+c^2)\left\{\sqrt{b^2+c^2}-b\right\}}$ Ans.

b) $\eta = \Delta E / E$

$$\eta = \Delta E/E = \frac{\frac{1}{2}mu_1^2 - \frac{1}{2}I_o\omega_2^2}{\frac{1}{2}mu_1^2} \longleftarrow I_o = m/3\,(b^2+c^2)$$

$$= \frac{\frac{1}{2}mu_1^2 - \frac{1}{2}\cdot\frac{m}{3}(b^2+c^2)\omega_2^2}{\frac{1}{2}mu_1^2}$$

$$= 1 - \frac{(b^2+c^2)}{3}\ \omega_2^2/u_1^2 \longleftarrow \omega_2 = \frac{3bu_1}{2(b^2+c^2)}$$

$$= 1 - \frac{\cancel{(b^2+c^2)}}{\cancel{3}} \cdot \frac{\frac{3\ \cancel{9}b^2\cancel{u_1^2}}{4(b^2+c^2)^{\cancel{2}}}}{\cancel{u_1^2}}$$

$$= 1 - \frac{3b^2}{4(b^2+c^2)}$$

when $b=c \longrightarrow \eta = 1 - \frac{3b^2}{4(b^2+b^2)} = 1 - \frac{3}{8} = 0.625$

$= 62.5\ \%$ Ans.