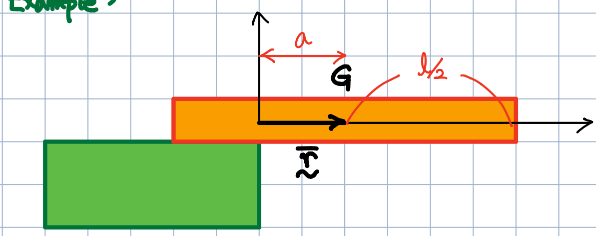
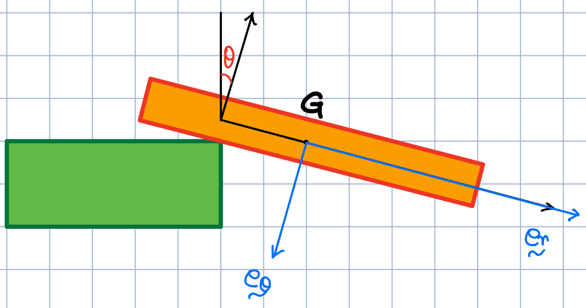
Example >



Find μ_s.

1. Kinematics



$$\bar{r} = r_G = a e_r$$

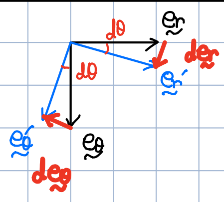
$$\dot{r}_G = a\dot{e}_r \quad = \dot{\theta} e_\theta$$

$$= a\dot{\theta} e_\theta$$

$$\ddot{r}_G = a\ddot{\theta} e_\theta + a\dot{\theta}\dot{e}_\theta \quad = -\dot{\theta} e_r$$

$$= a\ddot{\theta} e_\theta - a\dot{\theta}^2 e_r$$

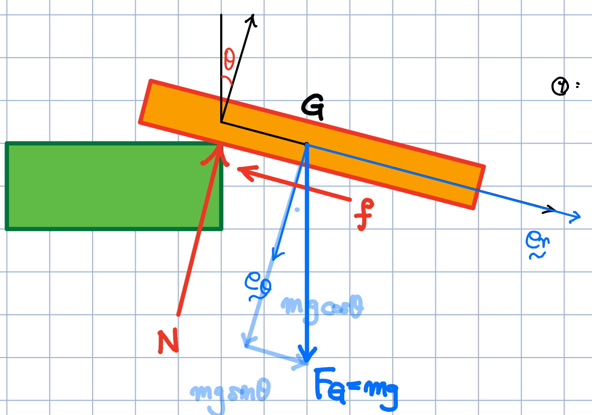
$$= a\alpha e_\theta - a\omega^2 e_r$$



$$de_r = d\theta \cdot e_\theta \qquad de_\theta = d\theta \cdot (-e_r)$$

$$\dot{e}_r = \dot{\theta} e_\theta \qquad \dot{e}_\theta = -\dot{\theta} e_r$$

2. FBD ⟶ EOM



① $\Sigma F_G = m a_G$ ② $\Sigma M_G = I_G \alpha$

①: $\Sigma F_G = \{N\cdot(-e_\theta)\} + \{f(-e_r)\} + \{mg\cos\theta\, e_\theta\} + \{mg\sin\theta\, e_r\} = m a_G = m(a\ddot{\theta} e_\theta - a\dot{\theta}^2 e_r)$

$-N + mg\cos\theta = ma\ddot{\theta}$ — a

$-f + mg\sin\theta = -ma\dot{\theta}^2$ — b

②: $\Sigma M_G = I_G \alpha$ (+ ↻)

$\Sigma M_G = Na = \frac{ml^2}{12} \cdot \ddot{\theta}$ — c

unknowns: $N, f, \dot{\theta}, \ddot{\theta}$ (4개)

eqs: a, b, c, constants (4개)

a&c: $-Na + mg\cos\theta \cdot a = ma^2\ddot{\theta}$

$$a\cancel{m}g\cos\theta = ma^2\ddot{\theta} + \frac{ml^2}{12}\ddot{\theta} = \cancel{m}\left(a^2 + \frac{l^2}{12}\right)\ddot{\theta}$$

$$\therefore \ddot{\theta} = \frac{ag\cos\theta}{a^2 + \frac{l^2}{12}}$$

b: $f = mg\sin\theta + ma\dot{\theta}^2$ $\longleftarrow$ $\ddot{\theta}\,d\theta = \dot{\theta}\,d\dot{\theta}$

$$\left(\frac{ag\cos\theta}{a^2+l^2/12}\right) d\theta = \dot{\theta}\,d\dot{\theta}$$

$$\frac{ag}{a^2+l^2/12}\underbrace{\int_0^\theta \cos\theta\,d\theta}_{=\sin\theta} = \frac{1}{2}\dot{\theta}^2$$

$$\dot{\theta}^2 = \frac{2ag\sin\theta}{a^2+l^2/12}$$

$$f = mg\sin\theta + ma\cdot\frac{2ag\sin\theta}{a^2+l^2/12}$$

$$= mg\sin\theta\left(1+\frac{2a^2}{a^2+l^2/12}\right)$$

$-N + mg\cos\theta = ma\ddot{\theta}$ $\longleftarrow$ $\therefore \ddot{\theta} = \dfrac{ag\cos\theta}{a^2+l^2/12}$

$$N = mg\cos\theta - ma\ddot{\theta}$$

$$= mg\cos\theta - ma\cdot\frac{ag\cos\theta}{a^2+l^2/12}$$

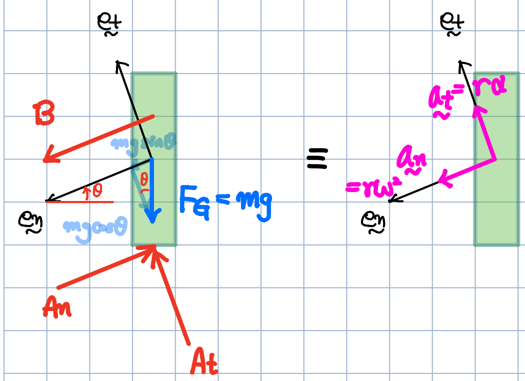
$$= mg\cos\theta\left(1-\frac{ma}{a^2+l^2/12}\right)$$

$$\therefore \mu_s = f/N = \frac{\cancel{mg}\sin\theta\left(1+\frac{2a^2}{a^2+l^2/12}\right)}{\cancel{mg}\cos\theta\left(1-\frac{ma}{a^2+l^2/12}\right)} = \frac{\left(1+\frac{2a^2}{a^2+l^2/12}\right)}{\left(1-\frac{ma}{a^2+l^2/12}\right)}\cdot\tan\theta$$

Ans.

· Vector !

FBD Kinematics



$$\underset{\sim}{a} = r\omega^2 \underset{\sim}{e_n} + r\alpha \underset{\sim}{e_t}$$

① $\Sigma \underset{\sim}{F_G} = m\underset{\sim}{a_G}$, ② $\Sigma M_G = I_G \alpha$

$$B\underset{\sim}{e_n} - A_n\underset{\sim}{e_n} + A_t\underset{\sim}{e_t} + mg\sin\theta\,\underset{\sim}{e_n} - mg\cos\theta\,\underset{\sim}{e_t} = mr\omega^2\underset{\sim}{e_n} + mr\alpha\underset{\sim}{e_t}$$

$$B - A_n + mg\sin\theta = mr\omega^2$$

$$A_t - mg\cos\theta = mr\alpha \longrightarrow \alpha = 14.81 - 6.54\cos\theta \text{ rad/s}^2$$

$$\downarrow$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$\frac{1}{2}\omega^2 = 14.81\theta - 6.54\sin\theta$$

$$\omega^2 = 29.6\theta - 13.08\sin\theta$$

Ans.