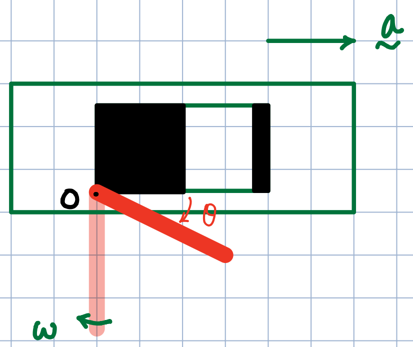
Problem 6.8



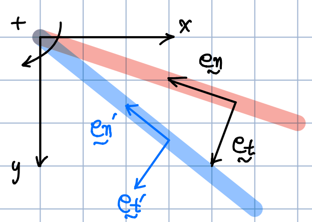
A car door is inadvertently left slightly open when the brakes are applied to give the car $\underset{\sim}{a}$.

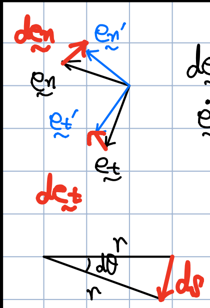
Derive expressions for the angular velocity (ω) as the door swings past the 90°.

Derive the hinge reactions for any θ.

The door mass : m
The mass center distance : $\bar{r}$
The radius of gyration about O : k_o

1. Coordinate (n-t)





$d\underset{\sim}{e_n} = d\theta\,(-\underset{\sim}{e_t})$ $\qquad d\underset{\sim}{e_t} = d\theta\,(\underset{\sim}{e_n})$

$\dot{\underset{\sim}{e_n}} = \dot{\theta}\,(-\underset{\sim}{e_t})$ $\qquad \dot{\underset{\sim}{e_t}} = \dot{\theta}\,(\underset{\sim}{e_n})$

$ds = r\,d\theta$ ← 미분 나누기,

$\frac{ds}{dt} = v = r\dot{\theta} = r\omega$

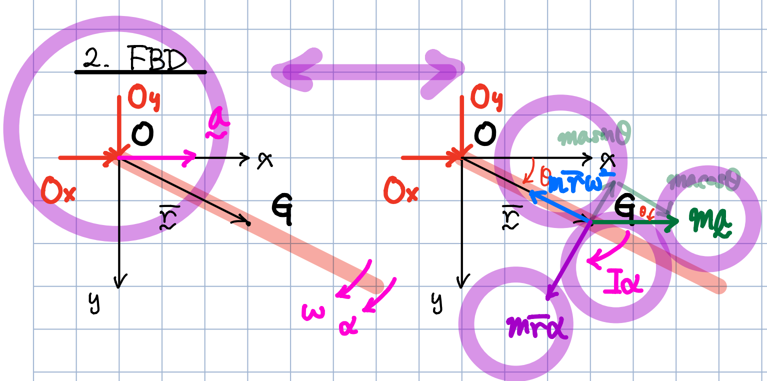
$\underset{\sim}{v} = v\underset{\sim}{e_t} = r\omega\underset{\sim}{e_t}$

$\underset{\sim}{a} = \dot{\underset{\sim}{v}} = \cancel{\dot{r}}\omega\underset{\sim}{e_t} + r\dot{\omega}\underset{\sim}{e_t} + r\omega\dot{\underset{\sim}{e_t}}$

r=const $\qquad = \dot{\theta}\underset{\sim}{e_n}$

$= r\alpha\underset{\sim}{e_t} + r\omega^2\underset{\sim}{e_n}$

2. FBD



$*$ angular velocity : ω

$\int_0^{\omega} \omega\,d\omega = \int_0^{\theta} \alpha\,d\theta \longrightarrow \alpha = ?$

3. EOM

$\Sigma \underset{\sim}{M_o} = \bar{I}\alpha + (m\bar{r}\alpha)\cdot(\bar{r}) - (ma\sin\theta)\cdot(\bar{r})$ ← $\bar{I} = m(k_o^2 - \bar{r}^2)$

$= m(k_o^2 - \cancel{\bar{r}^2})\alpha + \cancel{m\bar{r}^2\alpha} - ma\sin\theta\cdot\bar{r}$

$= \cancel{m}k_o^2\alpha - \cancel{m}a\sin\theta\cdot\bar{r} = 0$

$\alpha = \frac{a\bar{r}}{k_o^2}\sin\theta$

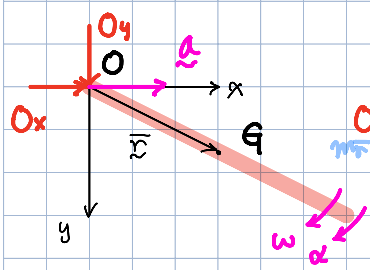
$$\int_0^\omega \omega\, d\omega = \int_0^\theta \frac{a\bar{r}}{k_0^2}\sin\theta\, d\theta$$

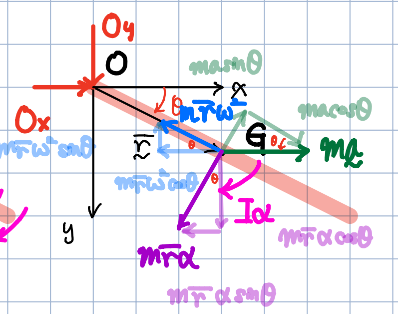
$$\frac{1}{2}\omega^2 = \frac{a\bar{r}}{k_0^2}\Big[-\cos\theta\Big]_0^\theta$$

$$= \frac{a\bar{r}}{k_0^2}(1-\cos\theta)$$

$$\boxed{\omega^2 = \frac{2a\bar{r}}{k_0^2}(1-\cos\theta)}$$

$$\omega = \frac{1}{k_0}\sqrt{2a\bar{r}(1-\cos\theta)} \quad \text{Ans.} \longrightarrow \quad \omega_{\theta=\pi/2} = \frac{1}{k_0}\sqrt{2a\bar{r}} \quad \text{Ans.}$$





$\Sigma F_x = ma_x$;

$$\boxed{\omega^2 = \frac{2a\bar{r}}{k_0^2}(1-\cos\theta) \qquad \alpha = \frac{a\bar{r}}{k_0^2}\sin\theta}$$

$$O_x = ma - m\bar{r}\omega^2\cos\theta - m\bar{r}\alpha\sin\theta$$

$$= ma - m\bar{r}\cdot\frac{2a\bar{r}}{k_0^2}(1-\cos\theta)\cos\theta - m\bar{r}\cdot\frac{a\bar{r}}{k_0^2}\sin^2\theta$$

$$= ma\left\{1 - \frac{2\bar{r}^2}{k_0^2}(\cos\theta-\cos^2\theta) - \frac{\bar{r}^2}{k_0^2}\underbrace{\sin^2\theta}_{=1-\cos^2\theta}\right\}$$

$$= ma\left\{1 - \frac{\bar{r}^2}{k_0^2}\underbrace{(2\cos\theta - 2\cos^2\theta + 1 - \cos^2\theta)}_{=1+2\cos\theta-3\cos^2\theta}\right\}$$

Ans.

$\Sigma F_y = ma_y$;

$$O_y = m\bar{r}\alpha\cos\theta - m\bar{r}\omega^2\sin\theta$$

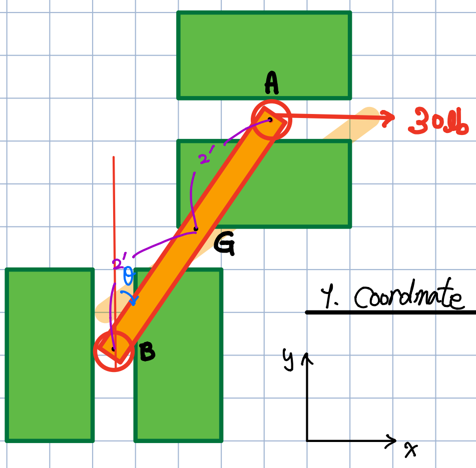
$$= m\bar{r}\,\frac{a\bar{r}}{k_0^2}\cos\theta\sin\theta - m\bar{r}\cdot\frac{2a\bar{r}}{k_0^2}(1-\cos\theta)\cdot\sin\theta$$

$$= \frac{ma\bar{r}^2}{k_0^2}\left\{\underbrace{\sin\theta\cos\theta - 2\sin\theta + 2\sin\theta\cos\theta}_{=3\sin\theta\cos\theta-2\sin\theta \;=\; (3\cos\theta-2)\sin\theta}\right\}$$

$$= \frac{ma\bar{r}^2}{k_0^2}(3\cos\theta-2)\sin\theta$$

Ans.

Example > 6.7



The slender bar AB weighs 60 lbs.

If the 30-lb force is applied at A with the bar initially at rest in the position for which $\theta = 30^\circ$,

Calculate the resulting angular acceleration of the bar (α)

and the force at A, B.

1. Coordinate

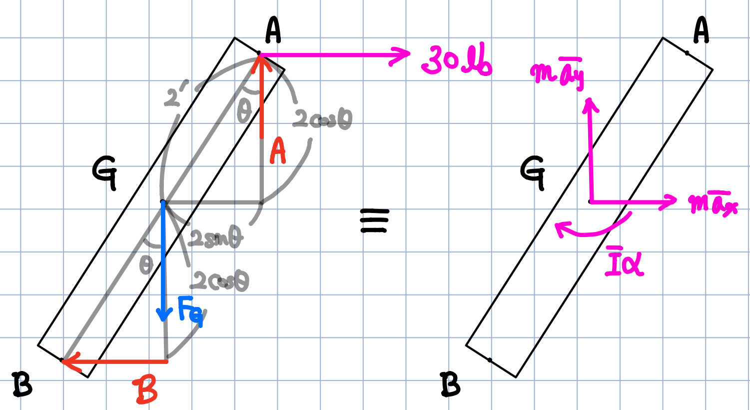
The bar: constrained motion

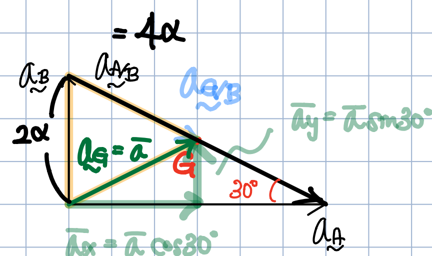
the mass-center acceleration ($\bar{a}$)

↕

the angular acceleration (α)

2. FBD





$\Sigma M_G = \bar{I}\alpha \quad (+\swarrow)$

$$\Sigma M_G = \boxed{B(2\cos\theta) - A(2\sin\theta) + 30(2\cos\theta)}$$

$$\bar{I}\alpha = \boxed{\left(\tfrac{1}{12} mL^2\right)\alpha = \left\{\tfrac{1}{12}\cdot\tfrac{60}{32.2}\cdot 4^2\right\}\alpha}$$

$a_A = a_B + a_{A/B}$

$\bar{a} = a_G = a_B + a_{G/B}$

$\bar{a}_x = \bar{a}\cos 30^\circ = 2\alpha\cos 30^\circ = 1.732\alpha \ ft/s^2$

$\bar{a}_y = \bar{a}\sin 30^\circ = 2\alpha\sin 30^\circ = 1\alpha \ ft/s^2$

$\Sigma F_x = m\bar{a}_x$

$= 30 - B = m\bar{a}_x$

$B = 30 - m\bar{a}_x$

$= 30 - \frac{60}{32.2}(1.732\alpha)$

$\Sigma F_y = m\bar{a}_y$

$= A - 60 = \frac{60}{32.2}(\alpha)$

$A = 60 + 60/32.2\,\alpha$

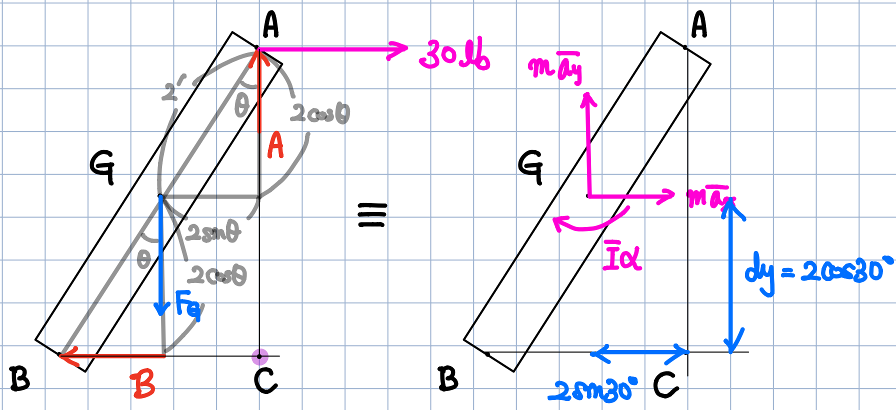
$$\left\{30 - 60/32.2\,(1.732\alpha)\right\}(2\cos\theta) - \left\{60 + 60/32.2\,\alpha\right\}(2\sin\theta) + 30(2\cos\theta)$$

$$= \left\{\tfrac{1}{12}\cdot 60/32.2\cdot 4^2\right\}\alpha$$

$\therefore \alpha = 4.42 \ rad/s^2$ Ans.

$A = 68.2 \ lb, \quad B = 15.74 \ lb$ Ans.

another way:



$\boxed{\Sigma M_G = \bar{I}\alpha \quad (+\circlearrowleft)}$

$\boxed{\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d}$

$\bar{a}_x = 1.732\alpha$

$\bar{a}_y = \alpha$

$$\Sigma M_C = (30\,lb)\cdot(4\cos 30°) - \overset{60\,lb}{F_G}(2\sin 30°)$$

$$\bar{I}\alpha + \Sigma m\bar{a}d = (\tfrac{1}{12})(\tfrac{60}{32.2})(4^2)\alpha + m a_x d_y + m a_y d_x$$

$$= (\tfrac{1}{12})(\tfrac{60}{32.2})(4^2)\alpha + (\tfrac{60}{32.2})(1.732\alpha)(2\cos 30°)$$

$$+ (\tfrac{60}{32.2})(\alpha)(2\sin 30°)$$

$$43.9 = 9.94\alpha$$

$\therefore \alpha = 4.42\ rad/s^2$ Ans.

$\boxed{\Sigma F_x = m\bar{a}_x}$

$\boxed{\Sigma F_y = m\bar{a}_y}$