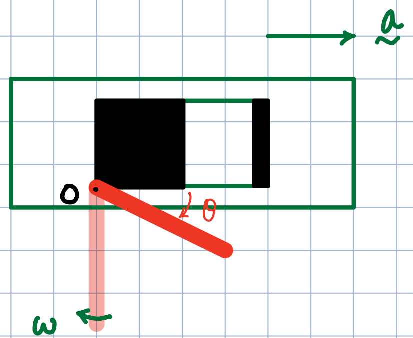
Problem 16.8



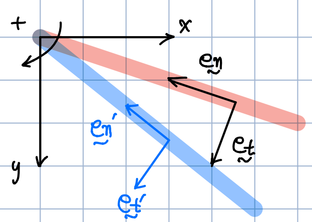
A car door is inadvertently left slightly open when the brakes are applied to give the car $\underset{\sim}{a}$.

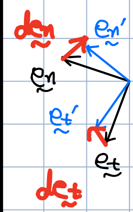
Derive expressions for the angular velocity (ω) as the door swings past the 90°.

Derive the hinge reactions for any θ.

The door mass : m
The mass center distance : $\bar{r}$
The radius of gyration about O : k_o

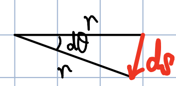
1. Coordinate (n-t)





$d\underset{\sim}{e_n} = d\theta(-\underset{\sim}{e_t})$ $\quad$ $d\underset{\sim}{e_t} = d\theta(\underset{\sim}{e_n})$

$\dot{\underset{\sim}{e_n}} = \dot\theta(-\underset{\sim}{e_t})$ $\quad$ $\dot{\underset{\sim}{e_t}} = \dot\theta(\underset{\sim}{e_n})$



$ds = r\,d\theta$ ← dt로 나누면,

$\frac{ds}{dt} = v = r\dot\theta = r\omega$

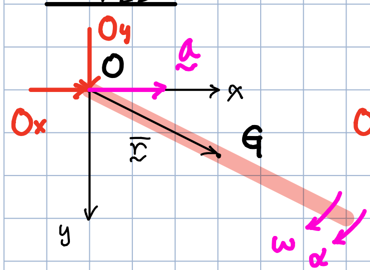
$\underset{\sim}{v} = v\underset{\sim}{e_t} = r\omega\underset{\sim}{e_t}$

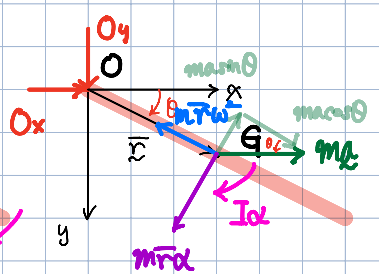
$\underset{\sim}{a} = \dot{\underset{\sim}{v}} = \cancel{\dot r}\omega\underset{\sim}{e_t} + r\dot\omega\underset{\sim}{e_t} + r\omega\dot{\underset{\sim}{e_t}}$

r=const $\quad$ $=\dot\theta\underset{\sim}{e_n}$

$= r\alpha\underset{\sim}{e_t} + r\omega^2\underset{\sim}{e_n}$

2. FBD





* angular velocity : ω

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta \longrightarrow \alpha = ?$

3. EOM

$\Sigma \underset{\sim}{M_o} = \bar{I}\alpha + (m\bar{r}\alpha)\cdot(\bar{r}) - (ma\sin\theta)\cdot(\bar{r})$ $\longleftarrow$ $\bar{I} = m(k_o^2 - \bar{r}^2)$

$= m(k_o^2 - \cancel{\bar{r}^2})\alpha + \cancel{m\bar{r}^2\alpha} - ma\sin\theta\cdot\bar{r}$

$= \cancel{m}k_o^2\alpha - \cancel{m}a\sin\theta\cdot\bar{r} = 0$

$\alpha = \frac{a\bar{r}}{k_o^2}\sin\theta$

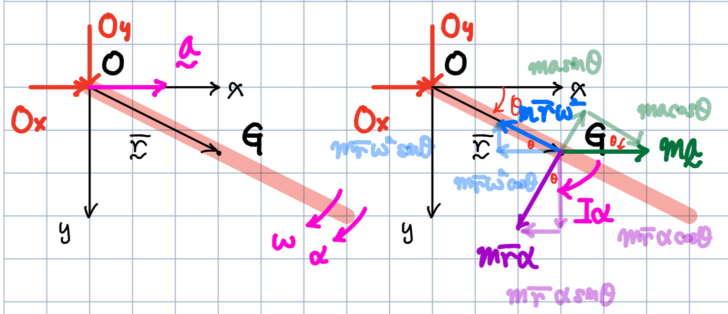
$$\int_0^{\omega} \omega\, d\omega = \int_0^{\theta} \frac{a\bar{r}}{k_o^2} \sin\theta \, d\theta$$

$$\frac{1}{2}\omega^2 = \frac{a\bar{r}}{k_o^2} \left[-\cos\theta\right]_0^{\theta}$$

$$= \frac{a\bar{r}}{k_o^2}(1-\cos\theta)$$

$$\boxed{\omega^2 = \frac{2a\bar{r}}{k_o^2}(1-\cos\theta)}$$

$$\underbrace{\omega = \frac{1}{k_o}\sqrt{2a\bar{r}(1-\cos\theta)}}\ \text{Ans.} \longrightarrow \underbrace{\omega_{\theta=\pi/2} = \frac{1}{k_o}\sqrt{2a\bar{r}}}\ \text{Ans.}$$



$\Sigma F_x = ma_x$;

$$O_x = ma - m\bar{r}\omega^2\cos\theta - m\bar{r}\alpha\sin\theta$$

$$\longleftarrow \boxed{\omega^2 = \frac{2a\bar{r}}{k_o^2}(1-\cos\theta) \qquad \alpha = \frac{a\bar{r}}{k_o^2}\sin\theta}$$

$$= ma - m\bar{r}\cdot\frac{2a\bar{r}}{k_o^2}(1-\cos\theta)\cos\theta - m\bar{r}\cdot\frac{a\bar{r}}{k_o^2}\sin^2\theta$$

$$= ma\left\{1 - \frac{2\bar{r}^2}{k_o^2}(\cos\theta - \cos^2\theta) - \frac{\bar{r}^2}{k_o^2}\underbrace{\sin^2\theta}_{=1-\cos^2\theta}\right\}$$

$$= ma\left\{1 - \frac{\bar{r}^2}{k_o^2}\underbrace{(2\cos\theta - 2\cos^2\theta + 1 - \cos^2\theta)}_{=1+2\cos\theta-3\cos^2\theta}\right\}$$

Ans.

$\Sigma F_y = ma_y$;

$$O_y = m\bar{r}\alpha\cos\theta - m\bar{r}\omega^2\sin\theta$$

$$= m\bar{r}\,\frac{a\bar{r}}{k_o^2}\cos\theta\sin\theta - m\bar{r}\cdot\frac{2a\bar{r}}{k_o^2}(1-\cos\theta)\cdot\sin\theta$$

$$= \frac{ma\bar{r}^2}{k_o^2}\left\{\underbrace{\sin\theta\cos\theta - 2\sin\theta + 2\sin\theta\cos\theta}_{=3\sin\theta\cos\theta - 2\sin\theta\ =\ (3\cos\theta-2)\sin\theta}\right\}$$

$$= \frac{ma\bar{r}^2}{k_o^2}(3\cos\theta - 2)\sin\theta$$

Ans.