Example > 6.4

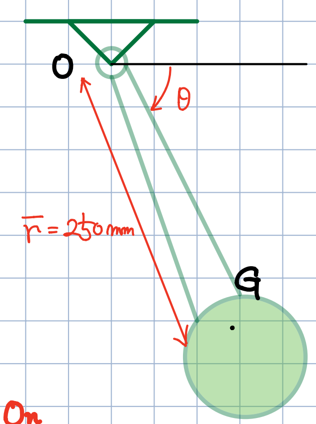
The pendulum has a mass of 7.5 kg with center of mass at G and has a radius of gyration about the pivot O of 295 mm.

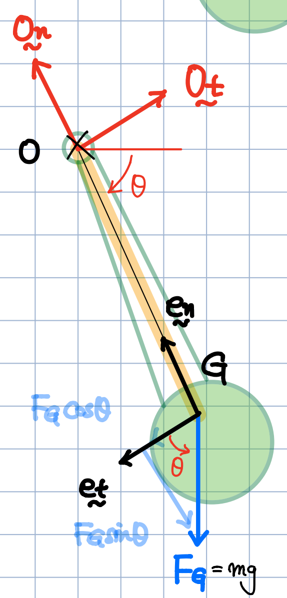
$k_O = \sqrt{I_O/m}$

If the pendulum is released from rest at $\theta = 0°$, determine the total force supported by the bearing at the instant when $\theta = 60°$

* Radius of gyration, $k = \sqrt{I/m} \rightarrow I = mk^2$

k is a measure of the distribution of mass of a given body about the axis.





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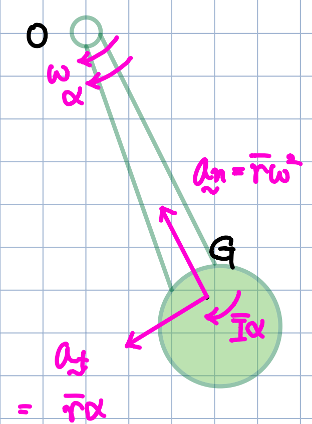


1. Translation : $\Sigma F = m\bar{a}$
2. Rotation : $\Sigma M_G = \dot{H}_G = \bar{I}\alpha$

$\Sigma M_O = I_O\alpha$

$\Sigma M_O = (mg\cos\theta)\cdot\bar{r} \quad (+\circlearrowright)$

$= (7.5)(9.81)\cos\theta \cdot (0.25\,m)$

$I_O\alpha = (mk_O^2)\alpha \leftarrow k_O = \sqrt{I_O/m}$

$= (7.5)(0.295)^2\alpha$

$\therefore \alpha = \dfrac{(7.5\,kg)(9.81\,m/s^2)\cos\theta\,(0.25\,m)}{(7.5\,kg)(0.295\,m)^2}$

$= 28.2\cos\theta$ rad/s²

$\omega\,d\omega = \alpha\,d\theta$

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta$

$\frac{1}{2}\omega^2 = \int_0^\theta (28.2\cos\theta)\,d\theta$

$= 28.2\sin\theta \leftarrow \theta = 60°$

$\therefore \omega^2 = 48.8\,(rad/s)^2$

$\Sigma F_n = m\bar{a}_n$
$= m\bar{r}\omega^2$

$\Sigma F_n = O_n - mg\sin\theta$
$= m\bar{r}\omega^2$

$\therefore O_n - (7.5)(9.81)\sin 60° = (7.5)(0.250)(48.8)$

$\therefore O_n = 155.2\,N$ Ans.

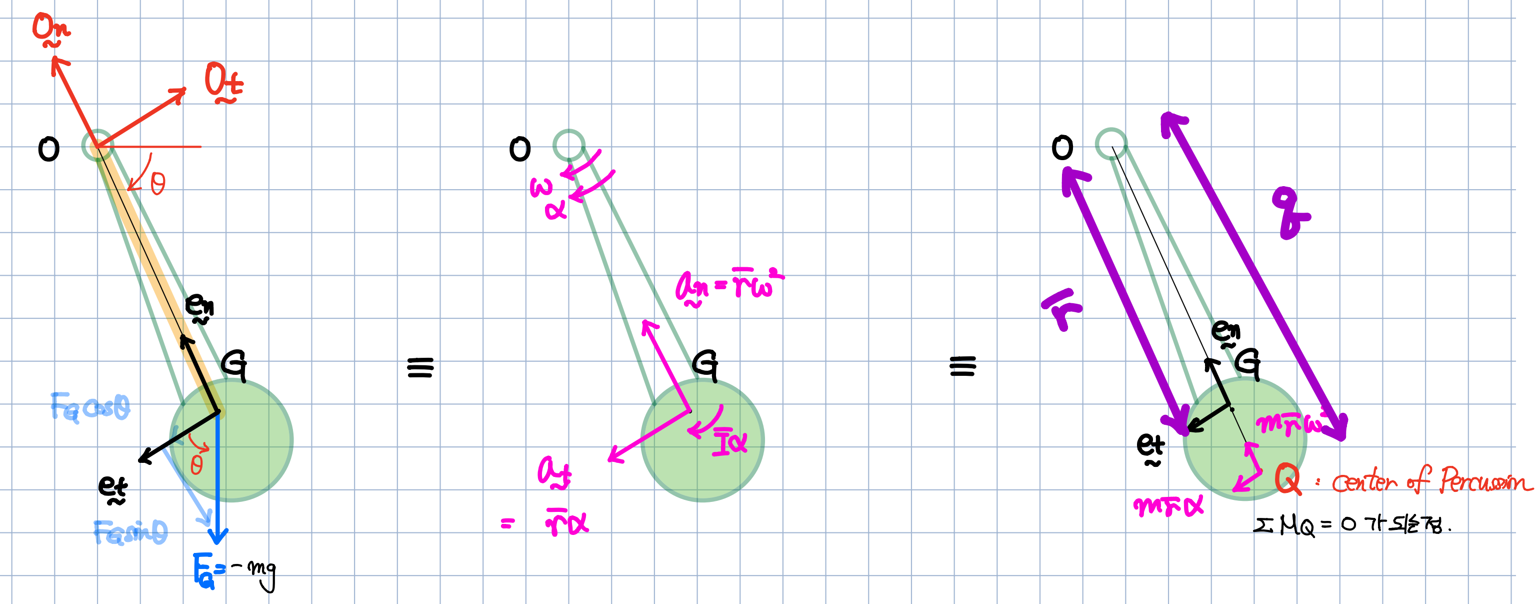
$\Sigma F_t = m\bar{a}_t$
$= m\bar{r}\alpha$

$\Sigma F_t = -O_t + mg\cos\theta$
$= m\bar{r}\alpha$

$\therefore -O_t + (7.5)(9.81)\cos 60° = (7.5)(0.250)(28.2\cos 60°)$

$\therefore O_t = 10.37\,N$

$O = \sqrt{O_n^2 + O_t^2} = \sqrt{155.2^2 + 10.37^2} = 155.6\,N$ Ans.



$$q = k_0^2 / \bar{r} \longrightarrow q = \frac{0.295^2}{0.250} = 0.348\,m$$

$$\Sigma M_Q = 0 \ ; \quad O_t\,(0.348) - (7.5)(9.81)\cos 60^\circ\,(0.348 - 0.250) = 0$$

$$\therefore O_t = 10.37\,N$$ Ans.