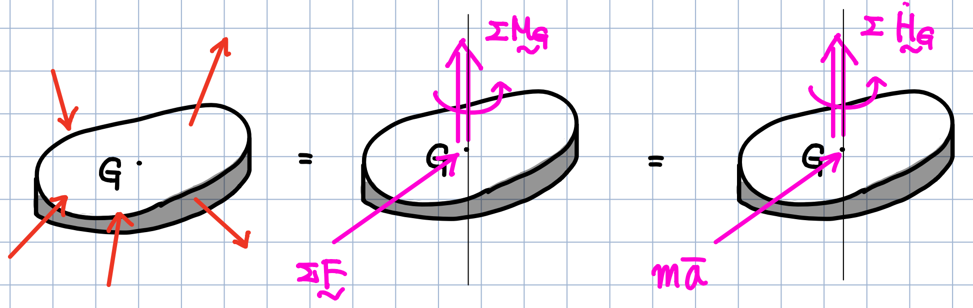
General EOM

1. Translation : $\Sigma \underset{\sim}{F} = m\underset{\sim}{\bar{a}}$
2. Rotation : $\Sigma \underset{\sim}{M}_G = \dot{\underset{\sim}{H}}_G$



Plane-Motion Equation (2-Dimension)

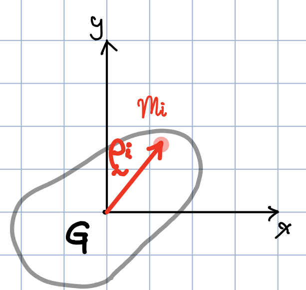
$\Sigma \underset{\sim}{M}_O = \underset{\sim}{r} \times \Sigma \underset{\sim}{F} = \underset{\sim}{r} \times m\dot{\underset{\sim}{v}}$

$\Sigma \underset{\sim}{H}_O = \underset{\sim}{r} \times m\underset{\sim}{v}$

$\Sigma \dot{\underset{\sim}{H}}_O = \dot{\underset{\sim}{r}} \times (m\underset{\sim}{v}) + \underset{\sim}{r} \times (m\dot{\underset{\sim}{v}})$
$= \cancel{\underset{\sim}{v} \times (m\underset{\sim}{v})} + \underset{\sim}{r} \times (m\dot{\underset{\sim}{v}})$
$= \Sigma M_O$

The angular momentum about the mass center : $\underset{\sim}{H}_G = \Sigma \underset{\sim}{\rho}_i \times m_i \dot{\underset{\sim}{\rho}}_i$

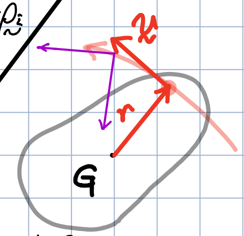
$$d\underset{\sim}{H}_G/dt = \cancel{\Sigma \dot{\underset{\sim}{\rho}}_i \times m_i \dot{\underset{\sim}{\rho}}_i} + \Sigma \underset{\sim}{\rho}_i \times m_i \ddot{\underset{\sim}{\rho}}_i$$

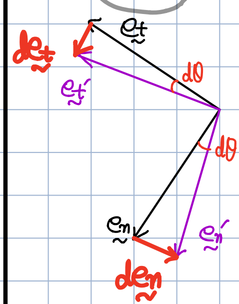


$\dot{\underset{\sim}{\rho}}_i = \underset{\sim}{\omega} \times \underset{\sim}{\rho}_i$ in rigid body

$\ddot{\underset{\sim}{\rho}}_i = \dot{\underset{\sim}{\omega}} \times \underset{\sim}{\rho}_i + \underset{\sim}{\omega} \times \dot{\underset{\sim}{\rho}}_i$

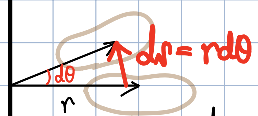
$= \dot{\underset{\sim}{\omega}} \times \underset{\sim}{\rho}_i + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{\rho}_i)$





$d\underset{\sim}{e}_t = d\theta(\underset{\sim}{e}_n)$ $\quad d\underset{\sim}{e}_n = d\theta(-\underset{\sim}{e}_t)$

$\dot{\underset{\sim}{e}}_t = \dot{\theta}\,\underset{\sim}{e}_n$ $\quad \dot{\underset{\sim}{e}}_n = -\dot{\theta}\,\underset{\sim}{e}_t$



$ds = r d\theta \leftarrow v = ds/dt$

$ds/dt = v = r d\theta/dt$

$\underset{\sim}{v} = v\underset{\sim}{e}_t$
$= r\omega\,\underset{\sim}{e}_t = \underset{\sim}{\omega} \times \underset{\sim}{r}$

$\underset{\sim}{H}_G = \Sigma \underset{\sim}{\rho}_i \times m_i \dot{\underset{\sim}{\rho}}_i$

$= \Sigma \underset{\sim}{\rho}_i \times m_i (\underset{\sim}{\omega} \times \underset{\sim}{\rho}_i)$ $\leftarrow \times : \sin 90° = 1$

$= \Sigma m_i \omega \rho_i^2 \underset{\sim}{k}$

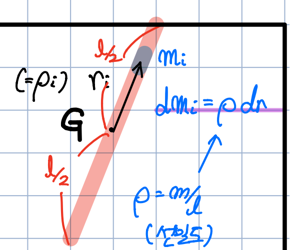
$d\underset{\sim}{H}_G/dt = \Sigma \underset{\sim}{\rho}_i \times m_i (\dot{\underset{\sim}{\omega}} \times \underset{\sim}{\rho}_i) + \cancel{\Sigma \underset{\sim}{\rho}_i \times m_i (\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{\rho}_i))}$

$\underset{\sim}{\rho}_i \parallel \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{\rho}_i)$

$= \Sigma m_i \alpha \rho_i^2 \underset{\sim}{k}$

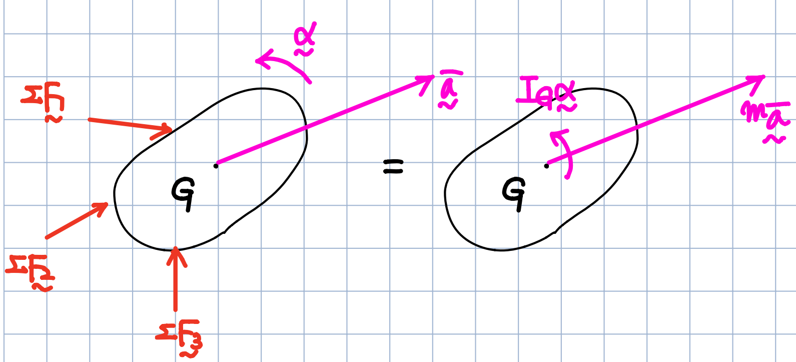
$\Sigma m_i \rho_i^2 = I_G = \int_A \rho r^2 dA \longrightarrow$

$I_G = \Sigma m_i r_i^2$
$= \int r_i^2\, dm$
$= \int r_i^2 \rho\, dr$
$= \frac{1}{3} r^3 \rho \Big]_{-l/2}^{l/2}$
$= \frac{1}{3}\rho\,(l^3/8 + l^3/8) = \frac{1}{3}\rho \cdot l^3/4 = \rho l^3/12 \leftarrow \rho = m/l$
$= (m/l) \cdot l^3/12$
$= ml^2/12$



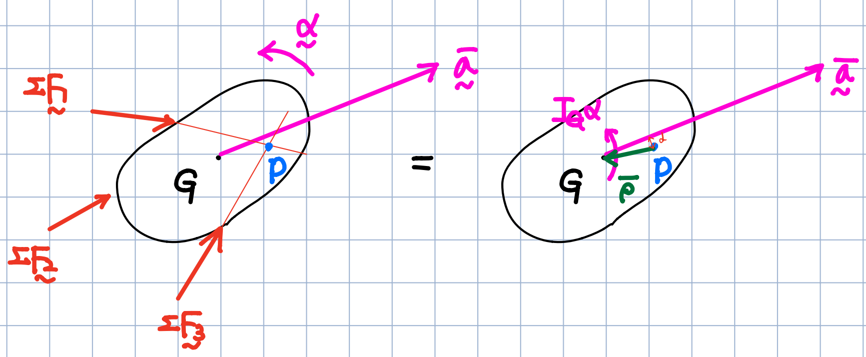
$$\boxed{\begin{aligned} \underset{\sim}{H}_G &= I_G \omega \underset{\sim}{k} \\ d\underset{\sim}{H}_G/dt &= I_G \alpha \underset{\sim}{k} \end{aligned}} \longrightarrow$$

1. Translation : $\Sigma \underset{\sim}{F} = m\underset{\sim}{\bar{a}}$
2. Rotation : $\Sigma \underset{\sim}{M}_G = \dot{\underset{\sim}{H}}_G$
$= I_G \alpha$



Alternative Moment Equation

P 점을 선택하면, 문제풀이가 간편해질 수 있다.



$$\Sigma F = m\bar{a}$$

$$\Sigma M_P = \dot{H}_G + \bar{\rho}_i \times m\bar{a}$$

$$= \bar{I}\alpha + m\bar{a}d$$

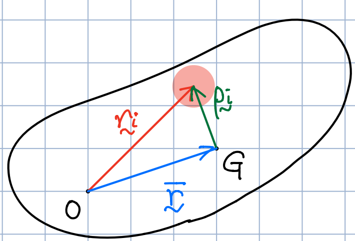
$$\Sigma M_P = (\dot{H}_P)_{rel} + \bar{\rho} \times m a_P$$

$$= I_P\alpha + \bar{\rho} \times m a_P$$

in case of fixed at O

$$\Sigma M_O = I_O\alpha$$

Transfer of axis Theorem



$$I_O = \Sigma m_i (r_i)\cdot(r_i)$$

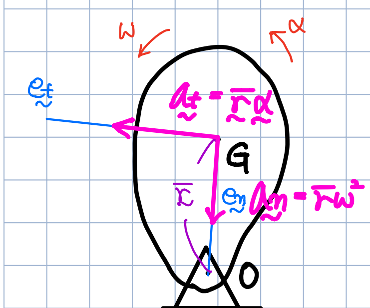
$$= \Sigma m_i (\bar{r}+\rho_i)\cdot(\bar{r}+\rho_i)$$

$$= \Sigma m_i \bar{r}\cdot\bar{r} + \Sigma m_i \rho_i\cdot\rho_i + \cancel{2\Sigma m_i \rho_i \cdot r_i} \rightarrow 0$$

$$= \boxed{M\bar{r}\cdot\bar{r} + I_G}$$

Ans.

Fixed-Axis Rotation



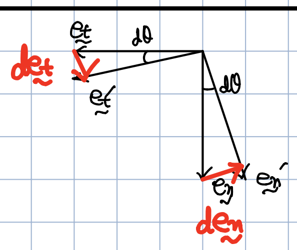
$$\Sigma F = m\bar{a}$$

$$\Sigma M_G = I_G\alpha$$

$$\Sigma M_O = I_O\alpha \leftarrow \Sigma M_O = \Sigma M_G + \bar{r}(m\bar{a}_t) \leftarrow \bar{a}_t = \bar{r}\alpha$$

$$= \Sigma M_G + m\bar{r}^2\alpha$$

$$= I_G\alpha + m\bar{r}^2\alpha = \boxed{(I_G + m\bar{r}^2)\alpha} = I_O\alpha$$



$$de_t = d\theta\, e_n, \quad de_n = d\theta \cdot (-e_t)$$

$$\dot{e}_t = \dot{\theta} e_n \qquad \dot{e}_n = -\dot{\theta} e_t$$

$$= \omega e_n$$

$$v = v e_t \leftarrow v = r\omega$$

$$= r\omega e_t$$

$$a = \cancel{\dot{r}}\omega e_t + r\dot{\omega} e_t + r\omega \dot{e}_t = r\omega^2 e_n + r\alpha e_t$$