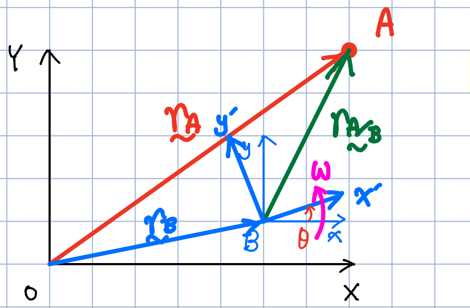
Motion Relative to Rotating Axes

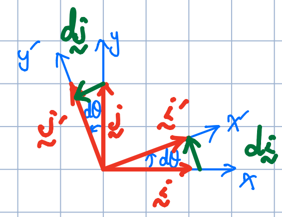


$$\boxed{\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}}$$

$$= \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

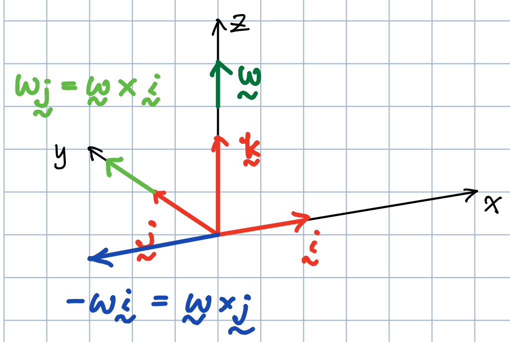
$\hookrightarrow \dot{x}\mathbf{i} + \cancel{x\dot{\mathbf{i}}}$

$$\dot{\mathbf{r}}_A = \mathbf{v}_A = \dot{\mathbf{r}}_B + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$$ —— translation.



$$\boxed{\begin{aligned} d\mathbf{i} &= d\theta\,\mathbf{j} \\ d\mathbf{j} &= d\theta\,(-\mathbf{i}) \end{aligned}}$$

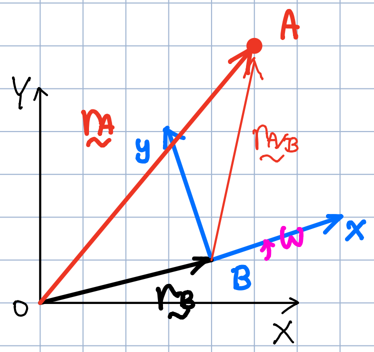
Time Derivatives of Unit Vectors



$$\dot{\mathbf{i}} = d\mathbf{i}/dt = d\theta\,\mathbf{j}/dt = \frac{d\theta}{dt}\mathbf{j} = \omega\mathbf{j} \longrightarrow \underline{\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i}}$$

$$\dot{\mathbf{j}} = d\mathbf{j}/dt = -d\theta\,\mathbf{i}/dt = -\frac{d\theta}{dt}\mathbf{i} = -\omega\mathbf{i} \longrightarrow \underline{\dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}}$$

Relative Velocity



$$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{AB}$$

$$= \underset{\sim}{r}_B + (x\underset{\sim}{i} + y\underset{\sim}{j})$$

$$\downarrow$$

$$\dot{\underset{\sim}{r}}_A = \dot{\underset{\sim}{r}}_B + \frac{d}{dt}(x\underset{\sim}{i} + y\underset{\sim}{j})$$

$$= \dot{\underset{\sim}{r}}_B + (\dot{x}\underset{\sim}{i} + x\dot{\underset{\sim}{i}}) + (\dot{y}\underset{\sim}{j} + y\dot{\underset{\sim}{j}}) \leftarrow \boxed{\dot{\underset{\sim}{i}} = \underset{\sim}{\omega} \times \underset{\sim}{i}\ ,\ \dot{\underset{\sim}{j}} = \underset{\sim}{\omega} \times \underset{\sim}{j}}$$

$$= \dot{\underset{\sim}{r}}_B + (\dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j}) + (\underset{\sim}{\omega} \times x\underset{\sim}{i} + \underset{\sim}{\omega} \times y\underset{\sim}{j})$$

$$= \underset{\sim}{\omega} \times (x\underset{\sim}{i} + y\underset{\sim}{j})$$

$$= \underset{\sim}{\omega} \times \underset{\sim}{r}$$

$$\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{\omega} \times \underset{\sim}{r} + \underset{\sim}{v}_{rel}$$

Ans.

Relative Acceleration

$$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B}$$

$$\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{\omega}\times\underset{\sim}{r} + \underset{\sim}{v}_{rel}$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \dot{\underset{\sim}{\omega}}\times\underset{\sim}{r} + \underset{\sim}{\omega}\times\dot{\underset{\sim}{r}} + \dot{\underset{\sim}{v}}_{rel}$$

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}} = \underbrace{(\dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j})}_{\underset{\sim}{v}_{rel}} + (x\dot{\underset{\sim}{i}} + y\dot{\underset{\sim}{j}}) \quad \leftarrow \quad \dot{\underset{\sim}{i}} = \underset{\sim}{\omega}\times\underset{\sim}{i}, \quad \dot{\underset{\sim}{j}} = \underset{\sim}{\omega}\times\underset{\sim}{j}$$

$$= \underset{\sim}{v}_{rel} + (\underset{\sim}{\omega}\times x\underset{\sim}{i} + \underset{\sim}{\omega}\times y\underset{\sim}{j})$$

$$= \underset{\sim}{v}_{rel} + \underset{\sim}{\omega}\times\underbrace{(x\underset{\sim}{i} + y\underset{\sim}{j})}_{\underset{\sim}{r}_{rel}}$$

$$= \underset{\sim}{v}_{rel} + \underset{\sim}{\omega}\times\underset{\sim}{r}_{rel}$$

$$\underset{\sim}{v}_{rel} = \dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j}$$

$$\dot{\underset{\sim}{v}}_{rel} = \underbrace{(\ddot{x}\underset{\sim}{i} + \ddot{y}\underset{\sim}{j})}_{=\underset{\sim}{a}_{rel}} + (\dot{x}\dot{\underset{\sim}{i}} + \dot{y}\dot{\underset{\sim}{j}}) \quad \leftarrow \quad \dot{\underset{\sim}{i}} = \underset{\sim}{\omega}\times\underset{\sim}{i}, \quad \dot{\underset{\sim}{j}} = \underset{\sim}{\omega}\times\underset{\sim}{j}$$

$$= \underset{\sim}{\omega}\times\dot{x}\underset{\sim}{i} + \underset{\sim}{\omega}\times\dot{y}\underset{\sim}{j}$$

$$= \underset{\sim}{\omega}\times(\dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j}) = \underset{\sim}{\omega}\times\underset{\sim}{v}_{rel}$$

$$= \underset{\sim}{a}_{rel} + \underset{\sim}{\omega}\times\underset{\sim}{v}_{rel}$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \dot{\underset{\sim}{\omega}}\times\underset{\sim}{r} + \underset{\sim}{\omega}\times\dot{\underset{\sim}{r}} + \dot{\underset{\sim}{v}}_{rel}$$

$$= \underset{\sim}{a}_B + \dot{\underset{\sim}{\omega}}\times\underset{\sim}{r} + \underset{\sim}{\omega}\times(\underset{\sim}{v}_{rel} + \underset{\sim}{\omega}\times\underset{\sim}{r}_{rel}) + \underset{\sim}{a}_{rel} + \underset{\sim}{\omega}\times\underset{\sim}{v}_{rel}$$

$$= \underset{\sim}{a}_B + \dot{\underset{\sim}{\omega}}\times\underset{\sim}{r} + \underset{\sim}{\omega}\times(\underset{\sim}{\omega}\times\underset{\sim}{r}_{rel}) + 2\underset{\sim}{\omega}\times\underset{\sim}{v}_{rel} + \underset{\sim}{a}_{rel}$$

Ans.

the Coriolis acceleration.