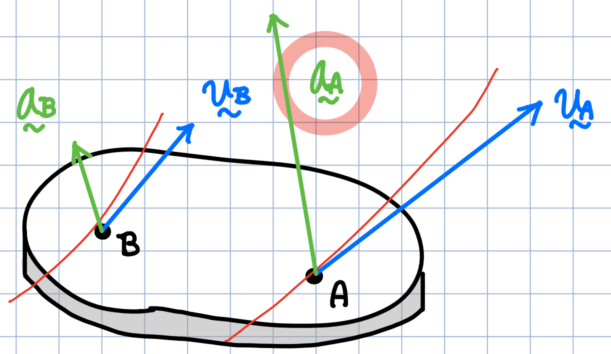
Relative Velocity : $v_A = v_B + v_{A/B}$

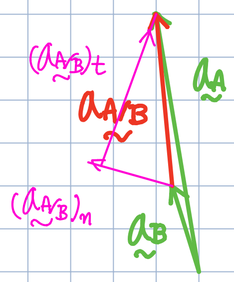
$$\downarrow$$

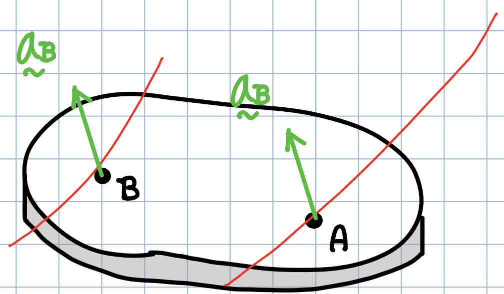
$$\dot{v}_A = \dot{v}_B + \dot{v}_{A/B}$$

Relative Acceleration : $a_A = a_B + a_{A/B}$

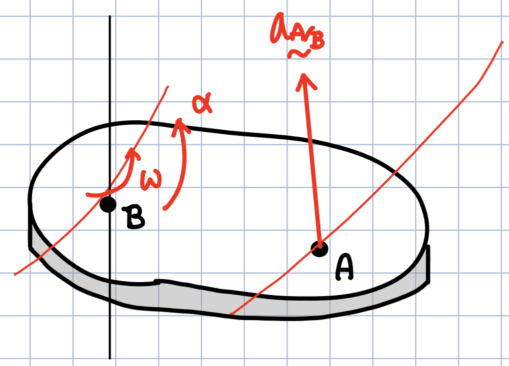
$$a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$$





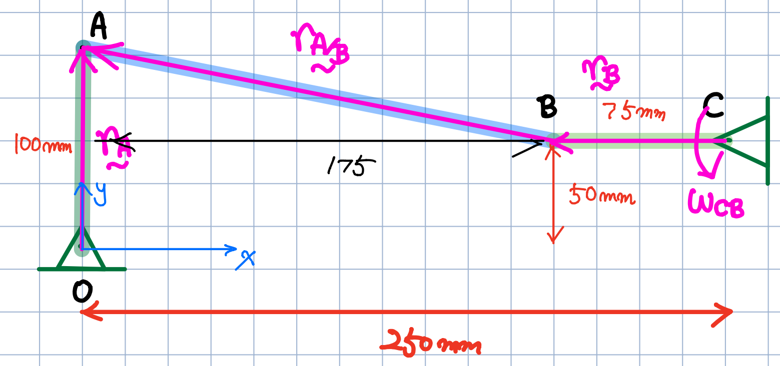


+



$$\boxed{\begin{aligned} v &= \omega \times r \\ a_n &= \omega \times (\omega \times r) \\ a_t &= \alpha \times r \end{aligned}}$$

Example > 5.8



$\omega_{CB} = 2\,rad/s$
(counterclockwise)

$\alpha_{OA},\ \alpha_{AB} = ?$

$\omega_{AB} = -6/7\ rad/s,\quad \omega_{OA} = -3/7\ rad/s$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B}$$

$$= \underset{\sim}{a}_B + (\underset{\sim}{a}_{A/B})_n + (\underset{\sim}{a}_{A/B})_t$$

$$\boxed{\begin{aligned} \underset{\sim}{v} &= \underset{\sim}{\omega} \times \underset{\sim}{r} \\ \underset{\sim}{a}_n &= \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) \\ \underset{\sim}{a}_t &= \underset{\sim}{\alpha} \times \underset{\sim}{r} \end{aligned}} \longrightarrow \boxed{\begin{aligned} \underset{\sim}{a} &= \underset{\sim}{a}_n + \underset{\sim}{a}_t \\ &= \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) + \underset{\sim}{\alpha} \times \underset{\sim}{r} \end{aligned}}$$

$$\underset{\sim}{a}_A = \underset{\sim}{\alpha}_{OA} \times \underset{\sim}{r}_A + \underset{\sim}{\omega}_{OA} \times (\underset{\sim}{\omega}_{OA} \times \underset{\sim}{r}_A)$$

$$= (\alpha_{OA}\,\underset{\sim}{k}) \times (100\,\underset{\sim}{j}) + (\omega_{OA}\,\underset{\sim}{k}) \times \{(\omega_{OA}\,\underset{\sim}{k}) \times (100\,\underset{\sim}{j})\}$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & \alpha_{OA} \\ 0 & 100 & 0 \end{vmatrix} \qquad \begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{OA} \\ 0 & 100 & 0 \end{vmatrix}$$

$$= -100\,\alpha_{OA}\,\underset{\sim}{i} + (\omega_{OA}\,\underset{\sim}{k}) \times (-100\,\omega_{OA}\,\underset{\sim}{i})$$

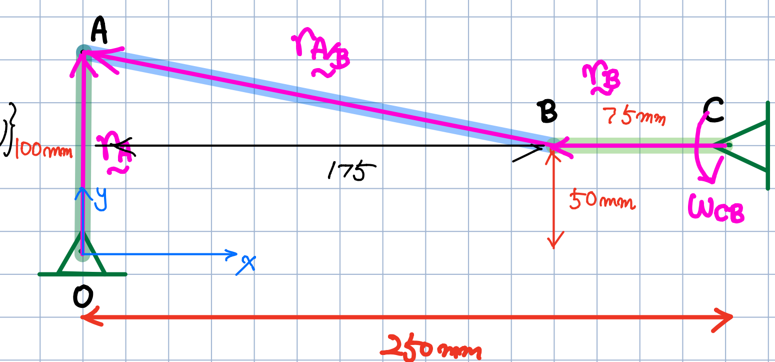
$$\begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{OA} \\ -100\,\omega_{OA} & 0 & 0 \end{vmatrix}$$

$$= -100\,\alpha_{OA}\,\underset{\sim}{i} - 100\,\omega_{OA}^2\,\underset{\sim}{j}$$

$$\underset{\sim}{a}_B = \underset{\sim}{\alpha}_{CB} \times \underset{\sim}{r}_B + \underset{\sim}{\omega}_{CB} \times (\underset{\sim}{\omega}_{CB} \times \underset{\sim}{r}_B)$$

$$= (\alpha_{CB}\,\underset{\sim}{k}) \times (-75\,\underset{\sim}{i}) + (\omega_{CB}\,\underset{\sim}{k}) \times \{(\omega_{CB}\,\underset{\sim}{k}) \times (-75\,\underset{\sim}{i})\}$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & \alpha_{CB} \\ -75 & 0 & 0 \end{vmatrix} \qquad \begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{CB} \\ -75 & 0 & 0 \end{vmatrix}$$



$$= -75\,\alpha_{CB}\,\underset{\sim}{j} + (\omega_{CB}\,\underset{\sim}{k}) \times (-75\,\omega_{CB}\,\underset{\sim}{j})$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{CB} \\ 0 & -75\,\omega_{CB} & 0 \end{vmatrix}$$

$$= -75\,\alpha_{CB}\,\underset{\sim}{j} + 75\,\omega_{CB}^2\,\underset{\sim}{i}$$

$$(\underset{\sim}{a}_{A/B})_n = \underset{\sim}{\omega}_{AB} \times (\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{A/B})$$

$$= (\omega_{AB}\,\underset{\sim}{k}) \times \{(\omega_{AB}\,\underset{\sim}{k}) \times (-175\,\underset{\sim}{i} + 50\,\underset{\sim}{j})\}$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{AB} \\ -175 & 50 & 0 \end{vmatrix} = -50\omega_{AB}\,\underset{\sim}{i} - 175\omega_{AB}\,\underset{\sim}{j}$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{AB} \\ -50\omega_{AB} & -175\omega_{AB} & 0 \end{vmatrix}$$

$$= 175\,\omega_{AB}^2\,\underset{\sim}{i} - 50\,\omega_{AB}^2\,\underset{\sim}{j}$$

$$(\underset{\sim}{a}_{AB})_t = \underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{A/B}$$

$$= \alpha_{AB}\,\underset{\sim}{k} \times (-175\,\underset{\sim}{i} + 50\,\underset{\sim}{j})$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & \alpha_{AB} \\ -175 & 50 & 0 \end{vmatrix}$$

$$= -50\,\alpha_{AB}\,\underset{\sim}{i} - 175\,\alpha_{AB}\,\underset{\sim}{j}$$

$$\boxed{\begin{aligned} \underset{\sim}{a}_A &= \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B} \\ &= \underset{\sim}{a}_B + (\underset{\sim}{a}_{A/B})_n + (\underset{\sim}{a}_{A/B})_t \end{aligned}}$$

$$-100\,\alpha_{OA}\,\underset{\sim}{i} \; -100\,\omega_{OA}^2\,\underset{\sim}{j} = -75\,\alpha_{CB}\,\underset{\sim}{j} + 75\,\omega_{CB}^2\,\underset{\sim}{i} + 175\,\omega_{AB}^2\,\underset{\sim}{i} - 50\,\omega_{AB}^2\,\underset{\sim}{j}$$
$$- 50\,\alpha_{AB}\,\underset{\sim}{i} - 175\,\alpha_{AB}\,\underset{\sim}{j}$$

$$= (75\,\omega_{CB}^2 + 175\,\omega_{AB}^2 - 50\,\alpha_{AB})\,\underset{\sim}{i} + (-75\,\alpha_{CB} - 50\,\omega_{AB}^2 - 175\,\alpha_{AB})\,\underset{\sim}{j}$$

$\omega_{AB} = -6/7$ rad/s , $\omega_{OA} = -3/7$ rad/s

$\omega_{CB} = 2$ rad/s (const) $\longrightarrow$ $\alpha_{CB} = 0$

$$-100\,\omega_{OA}^2 = -75\,\alpha_{CB} - 50\,\omega_{AB}^2 - 175\,\alpha_{AB}$$

$$-100\cdot(-3/7)^2 = -75\,\cancel{\alpha_{CB}} - 50\,(-6/7)^2 - 175\,\alpha_{AB}$$

$$175\,\alpha_{AB} = 100\,(-3/7)^2 - 50\,(-6/7)^2$$
$$= -18.37$$

$$\alpha_{AB} = -0.1050 \text{ rad/s}^2$$

Ans.

$$-100\cdot\alpha_{OA} = 75\,\omega_{CB}^2 + 175\,\omega_{AB}^2 - 50\,\alpha_{AB}$$

$$= 75\,(2)^2 + 175\,(-6/7)^2 - 50\,(-0.1050)$$

$$\therefore \alpha_{OA} = -4.34 \text{ rad/s}^2$$

Ans.