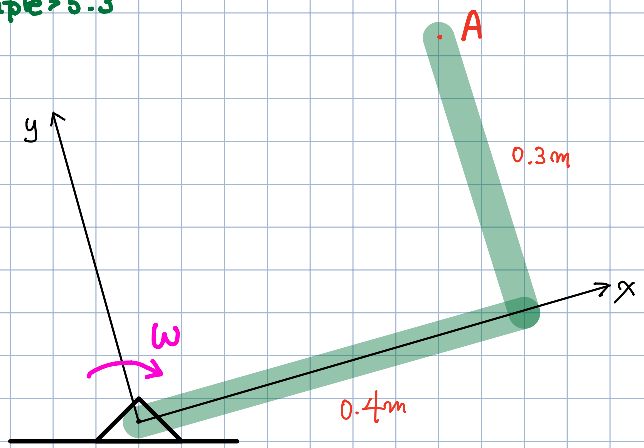
Example > 5.3



The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s²

Determine velocity & acceleration of point A when $\omega = 2$ rad/s ?

$$\boxed{\begin{aligned} \underset{\sim}{v} &= \underset{\sim}{\omega} \times \underset{\sim}{r} \\ \underset{\sim}{a_n} &= \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) \\ \underset{\sim}{a_t} &= \underset{\sim}{\alpha} \times \underset{\sim}{r} \\ \underset{\sim}{a} &= \underset{\sim}{a_n} + \underset{\sim}{a_t} \end{aligned}}$$

rotates clockwise : $\underset{\sim}{\omega} = -2\underset{\sim}{k}$ rad/s

$\underset{\sim}{\alpha} = 4\underset{\sim}{k}$ rad/s²

$\underset{\sim}{r_A} = 0.4\underset{\sim}{i} + 0.3\underset{\sim}{j}$

$$\underset{\sim}{v_A} = \underset{\sim}{\omega} \times \underset{\sim}{r} = (-2\underset{\sim}{k}) \times (0.4\underset{\sim}{i} + 0.3\underset{\sim}{j}) = \begin{vmatrix} i & j & k \\ 0 & 0 & -2 \\ 0.4 & 0.3 & 0 \end{vmatrix} = \underset{\sim\sim\sim}{0.6\underset{\sim}{i} - 0.8\underset{\sim}{j} \ \text{m/s}} \ \text{Ans.}$$

$$|\underset{\sim}{v}| = \sqrt{0.6^2 + 0.8^2} = \underset{\sim\sim}{1 \text{ m/s}} \ \text{Ans.}$$

$$\underset{\sim}{a} = \underset{\sim}{a_n} + \underset{\sim}{a_t}$$

$$\underset{\sim}{a_n} = \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) = \begin{vmatrix} i & j & k \\ 0 & 0 & -2 \\ 0.6 & -0.8 & 0 \end{vmatrix} = \underset{\sim\sim\sim}{-1.6\underset{\sim}{i} - 1.2\underset{\sim}{j}}$$

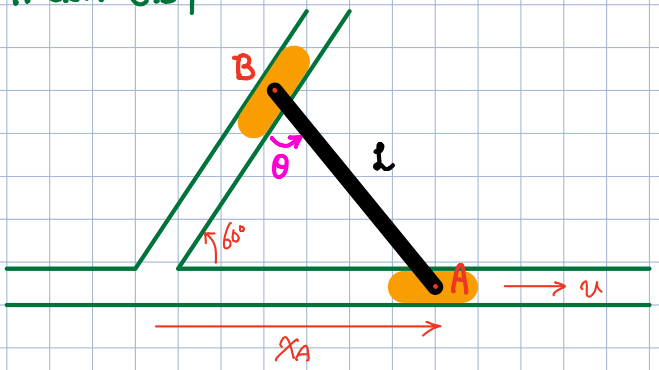
$$\underset{\sim}{a_t} = \underset{\sim}{\alpha} \times \underset{\sim}{r} = \begin{vmatrix} i & j & k \\ 0 & 0 & 4 \\ 0.4 & 0.3 & 0 \end{vmatrix} = \underset{\sim\sim\sim}{-1.2\underset{\sim}{i} + 1.6j}$$

$$\therefore \underset{\sim\sim\sim}{\underset{\sim}{a} = -2.8\underset{\sim}{i} + 0.4\underset{\sim}{j}} \ \text{Ans.} \longrightarrow |\underset{\sim}{a}| = \sqrt{2.8^2 + 0.4^2} = \underset{\sim\sim\sim}{2.83 \text{ m/s}} \ \text{Ans.}$$

Absolute Motion

The geometric relations $\longrightarrow$ define the configuration of the body.

Problem > 5.34



Determine the angular velocity ω of the bar AB in terms of x_A

$\dot{\theta}$

$$\frac{x_A}{\sin\theta} = \frac{L}{\sin 60^\circ} \longrightarrow x_A = \frac{L\sin\theta}{\sin 60^\circ} = \frac{2}{\sqrt{3}}L\sin\theta$$

$$\dot{x}_A = \frac{2}{\sqrt{3}}L\cos\theta\cdot\dot{\theta}$$

u ω

$$\sin\theta = \sin 60^\circ \cdot x_A/L$$
$$= \frac{\sqrt{3}}{2}x_A/L$$

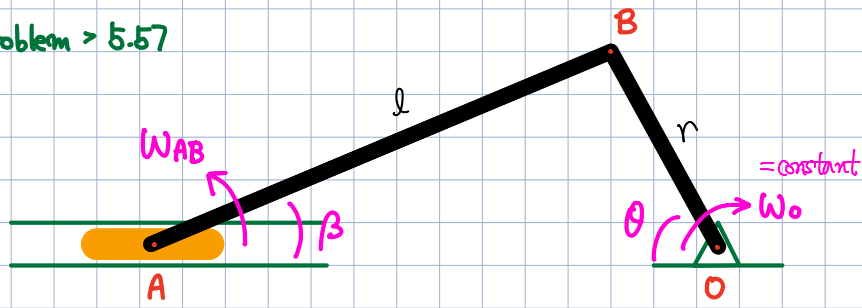
$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{3}{4}x_A^2/L^2}$$

$$\dot{\theta} = \omega = \frac{\sqrt{3}u}{2L\cos\theta} = \frac{\sqrt{3}u}{2L}\cdot\frac{1}{\sqrt{1-\frac{3}{4}\cdot x_A^2/L^2}}$$

Ans.

Problem > 5.57



The slider crank

Determine the angular velocity (ω_{AB})
the angular acceleration (α_{AB})
in terms of θ.

$$\boxed{\frac{\sin\theta}{l} = \frac{\sin\beta}{r}} \longrightarrow \sin\beta = \frac{r}{l}\sin\theta$$

$$\cos\beta = \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}$$

$$l\sin\beta = r\sin\theta$$

$$l\cos\beta\,\underbrace{\dot\beta}_{W_{AB}} = r\cos\theta\,\underbrace{\dot\theta}_{W_o}$$

$$W_{AB} = \dot\beta = \frac{r\cos\theta\cdot W_o}{l\cdot\cos\beta} = \frac{r\cos\theta\cdot\omega_o}{l\cdot\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}$$

Ans.

$$l(-\sin\beta\cdot\dot\beta)\dot\beta + l\cos\beta\,\underbrace{\ddot\beta}_{\alpha_{AB}} = r(-\sin\theta\,\dot\theta)\dot\theta + r\cos\theta\,\cancel{\ddot\theta}$$

$\because W_o = \text{const}$

$$\alpha_{AB} = \ddot\beta = \frac{l\dot\beta^2\sin\beta - r\dot\theta^2\sin\theta}{l\cos\beta}$$

$$= \frac{\cancel{l}\cdot\frac{r^2\cos^2\theta\,\omega_o^2}{l^{\cancel{2}}(1-\frac{r^2}{l^2}\sin^2\theta)}\sin\beta - r\,\omega_o^2\sin\theta}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}$$

$$= \frac{\frac{r^2\cos^2\theta\,\omega_o^2}{l(1-\frac{r^2}{l^2}\sin^2\theta)}\cdot\frac{r}{l}\sin\theta - \frac{r\omega_o^2\sin\theta}{l}\cdot l}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}$$

$$= \frac{r\omega_o^2\sin\theta}{l}\cdot\frac{\frac{r^2\cos^2\theta}{l(1-\frac{r^2}{l^2}\sin^2\theta)} - l}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}$$

$$= \frac{r\omega_o^2\sin\theta}{l}\cdot\frac{\frac{r^2}{l^2}\cos^2\theta - 1\cdot(1-\frac{r^2}{l^2}\sin^2\theta)}{(1-\frac{r^2}{l^2}\sin^2\theta)^{3/2}}$$

$$= \frac{r\omega_o^2\sin\theta}{l}\cdot\frac{\frac{r^2}{l^2} - 1}{(1-\frac{r^2}{l^2}\sin^2\theta)^{3/2}}$$

Ans.