Dynamics of Particles

1. Introduction to Dynamics : Newton's Laws, units, dimensions
2. Kinematics of Particles
3. Kinetics of Particles
4. Kinetics of Systems of Particles

Dynamics of Rigid Bodies

1. Plane Kinematics of Rigid Bodies
2. Plane Kinetics of Rigid Bodies
3. 3D Dynamics of Rigid Bodies
4. Vibration

Chapter 5. Kinematics of Rigid Bodies

Plane Motion (Absolute-Motion Analysis)

Translation : Rectilinear translation, Curvilinear translation

Rotation : Fixed-axis rotation, General plane motion

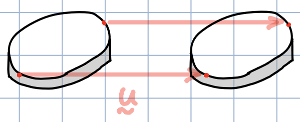
Relative Velocity and Acceleration (Relative-Motion Analysis)

Motion Relative to Rotating Axes (Rotation Coordinate System)

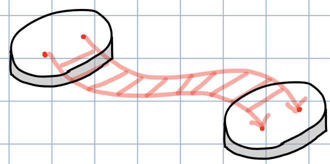
Types of Rigid-Bodies Plane Motion

Translation

1. Rectilinear Motion

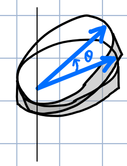


2. Curvilinear Motion

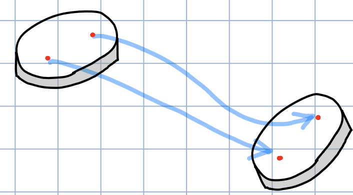


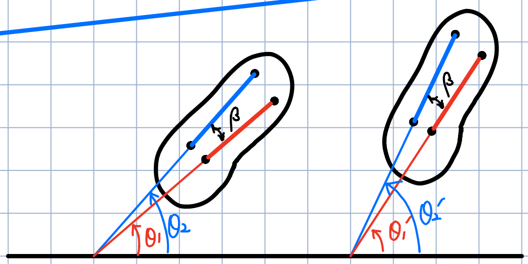
Rotation

3. Fixed-axis Rotation



4. General Plane Motion





fixed reference : $\boxed{\begin{aligned}\theta_2 &= \theta_1 + \beta \\ \theta_2' &= \theta_1' + \beta\end{aligned}}$

$\theta_1' - \theta_1 = \Delta\theta_1$

$\theta_2' - \theta_2 = \Delta\theta_2$

$$\begin{aligned}\Delta\theta_2 &= \theta_2' - \theta_2 \\ &= (\theta_1' + \beta) - (\theta_1 + \beta) \\ &= \theta_1' - \theta_1 \\ &= \Delta\theta_1\end{aligned}$$

$\therefore \Delta\theta_2 = \Delta\theta_1$

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$\longrightarrow$

$$a\,ds = u\,du$$

$$\boxed{\begin{aligned}\alpha\, d\theta &= \omega\, d\omega \\ \ddot{\theta}\, d\theta &= \dot{\theta}\, d\dot{\theta}\end{aligned}}$$

if α = constant

$$\begin{aligned}\alpha &= \frac{d\omega}{dt} \\ \alpha\, dt &= d\omega \\ \int \alpha\, dt &= \int d\omega \\ \alpha \int_0^t dt &= \int_{\omega_0}^{\omega} d\omega \\ \alpha t &= \omega - \omega_0\end{aligned}$$

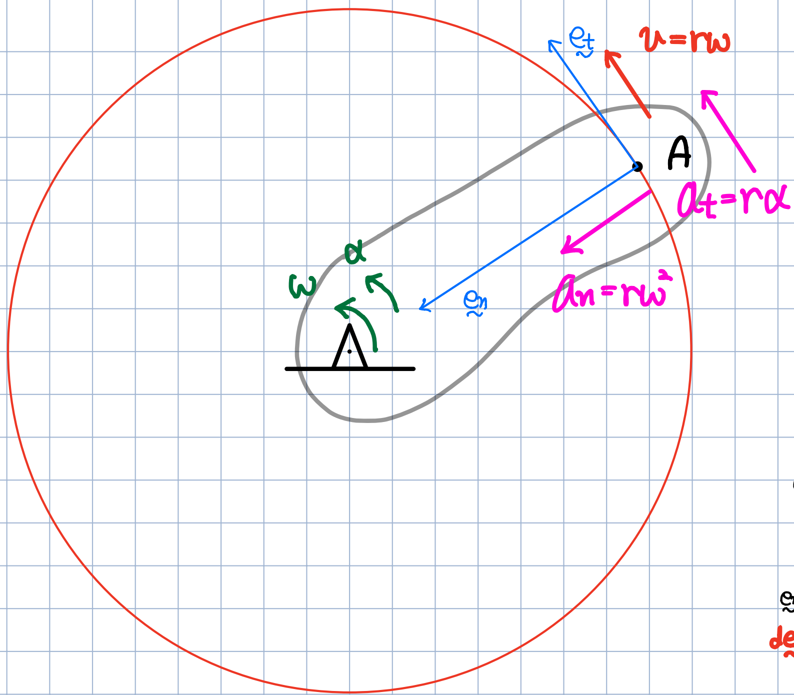
$\therefore \omega = \omega_0 + \alpha t$

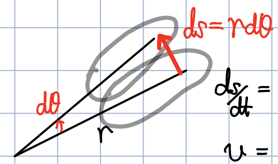
$$\begin{aligned}\omega\, d\omega &= \alpha\, d\theta \\ \int \omega\, d\omega &= \int \alpha\, d\theta \\ \int_{\omega_0}^{\omega} \omega\, d\omega &= \alpha \int_{\theta_0}^{\theta} d\theta \\ \frac{1}{2}\omega^2 \Big]_{\omega_0}^{\omega} &= \alpha(\theta - \theta_0)\end{aligned}$$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$$\begin{aligned}\omega &= \frac{d\theta}{dt} \\ d\theta &= \omega\, dt \\ \int_{\theta_0}^{\theta} d\theta &= \int_0^t \omega\, dt \\ &= \int_0^t \omega_0 + \alpha t\, dt \\ \theta - \theta_0 &= \omega_0 t + \frac{1}{2}\alpha t^2\end{aligned}$$

$\therefore \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$





$\leftarrow v = ds/dt$

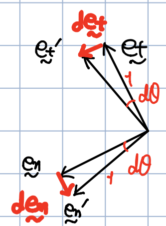
$$\frac{ds}{dt} = r\,\frac{d\theta}{dt}$$

$$v = r\dot{\theta} = r\omega$$

$$\mathbf{v} = v\,\mathbf{e}_t$$
$$= r\omega\,\mathbf{e}_t$$

$$\mathbf{a} = \dot{\mathbf{v}} = \cancel{\dot{r}}\omega\,\mathbf{e}_t + r\dot{\omega}\,\mathbf{e}_t + r\omega\,\dot{\mathbf{e}}_t$$

∵ rigid body



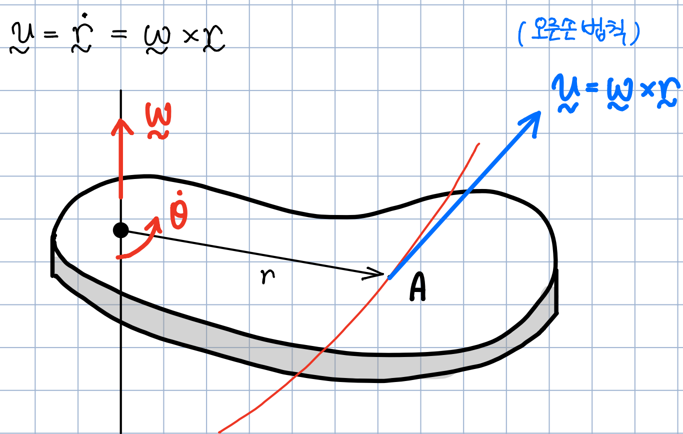
$d\mathbf{e}_t = d\theta\,\mathbf{e}_n$ $\qquad$ $d\mathbf{e}_n = d\theta\,(-\mathbf{e}_t)$

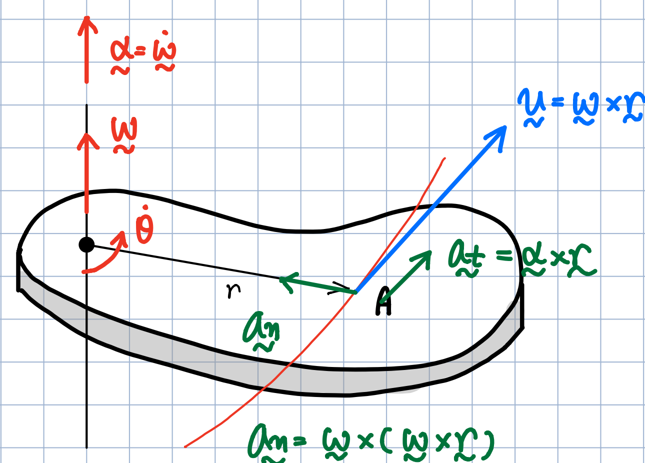
$\dot{\mathbf{e}}_t = \dot{\theta}\,\mathbf{e}_n$ $\qquad$ $\dot{\mathbf{e}}_n = -\dot{\theta}\,\mathbf{e}_t$

$$\mathbf{a} = r\dot{\omega}\,\mathbf{e}_t + r\omega(\dot{\theta}\,\mathbf{e}_n)$$
$$= r\omega^2\,\mathbf{e}_n + r\alpha\,\mathbf{e}_t$$

The vector notation

$$\mathbf{v} = \dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$$





$$\boxed{\begin{aligned} \mathbf{v} &= \boldsymbol{\omega} \times \mathbf{r} \\ \mathbf{a}_n &= \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \\ \mathbf{a}_t &= \boldsymbol{\alpha} \times \mathbf{r} \\ \mathbf{a} &= \mathbf{a}_n + \mathbf{a}_t \end{aligned}}$$

$\leftarrow a = \sqrt{a_n^2 + a_t^2}$