

$$y \frac{dy}{dx} = 2y - x$$

$$\frac{dy}{dx} = 2 - \frac{x}{y} \quad \boxed{\frac{x}{y} = u}$$

$$dy = (2 - u) dx \quad x = uy$$
$$\boxed{x' = u + y u'}$$

$$1 = (2 - u)(u + u'y)$$
$$= 2u + 2u'y - u^2 - uu'y$$

$$(u^2 - 2u + 1) = u'(2y - uy)$$

$$(u-1)^2 = u'(2-u)y$$

$$\frac{1}{y} = \frac{2-u}{(u-1)^2} u' \quad \boxed{u-1=t}$$
$$\underline{dt = du}$$

$$\ln|y| + C = \int \frac{-t+1}{t^2} du$$

$$= \int -\frac{1}{t} + \frac{1}{t^2} dt$$

$$= -\ln|u-1| - \frac{1}{u-1}$$

$$\ln|y| + C = -\ln\left|\frac{x}{y}-1\right| - \frac{1}{\frac{x}{y}-1}$$

$$\ln|y| + \ln\left|\frac{x}{y}-1\right| = -\frac{y}{x-y} + C'$$

$$\ln|x-y| = \ln e^{\frac{-y}{y-x} \cdot C'}$$

$$x-y = e^{\frac{-y}{y-x} \cdot C'}$$

$$x - C \cdot e^{\frac{-y}{y-x}} = y$$