p107 3.5 예제 5.3(2)

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$(*)$: $y^{(3)} - 4y^{(2)} - 3y^{(1)} = 3\cos x$

step 1) y_h

$\lambda^3 - 4\lambda^2 - 3\lambda = \lambda(\lambda^2 - 4\lambda - 3)$

$\lambda = 0,\ 2 \pm \sqrt{4+3}$

$\therefore y_h = c_1 + c_2 e^{(2+\sqrt{7})x} + c_3 e^{(2-\sqrt{7})x}$

step 2) y_p

$y_1 = 1,\ y_2 = e^{(2+\sqrt{7})x},\ y_3 = e^{(2-\sqrt{7})x}$

$\therefore y_p = u_1 + u_2 e^{(2+\sqrt{7})x} + u_3 e^{(2-\sqrt{7})x}$

$$W = \begin{vmatrix} 1 & e^{(2+\sqrt{7})x} & e^{(2-\sqrt{7})x} \\ 0 & (2+\sqrt{7})e^{(2+\sqrt{7})x} & (2-\sqrt{7})e^{(2-\sqrt{7})x} \\ 0 & (2+\sqrt{7})^2 e^{(2+\sqrt{7})x} & (2-\sqrt{7})^2 e^{(2-\sqrt{7})x} \end{vmatrix} = (2+\sqrt{7})(2-\sqrt{7})^2 e^{4x} - (2-\sqrt{7})(2+\sqrt{7})^2 e^{4x} = 6\sqrt{7}e^{4x}$$

$r(x) = 3\cos x$

$$W_1 = \begin{vmatrix} 0 & e^{(2+\sqrt{7})x} & e^{(2-\sqrt{7})x} \\ 0 & (2+\sqrt{7})e^{(2+\sqrt{7})x} & (2-\sqrt{7})e^{(2-\sqrt{7})x} \\ 3\cos x & (2+\sqrt{7})^2 e^{(2+\sqrt{7})x} & (2-\sqrt{7})^2 e^{(2-\sqrt{7})x} \end{vmatrix} = 3\cos x \times (2-\sqrt{7})e^{4x} - 3\cos x \times (2+\sqrt{7})e^{4x} = -6\sqrt{7}\cos x \times e^{4x}$$

$$W_2 = \begin{vmatrix} 1 & 0 & e^{(2-\sqrt{7})x} \\ 0 & 0 & (2-\sqrt{7})e^{(2-\sqrt{7})x} \\ 0 & 3\cos x & (2-\sqrt{7})^2 e^{(2-\sqrt{7})x} \end{vmatrix} = -3\cos x \times (2-\sqrt{7})e^{(2-\sqrt{7})x}$$

$$W_3 = \begin{vmatrix} 1 & e^{(2+\sqrt{7})x} & 0 \\ 0 & (2+\sqrt{7})e^{(2+\sqrt{7})x} & 0 \\ 0 & (2+\sqrt{7})^2 e^{(2+\sqrt{7})x} & 3\cos x \end{vmatrix} = 3\cos x \times (2+\sqrt{7})e^{(2+\sqrt{7})x}$$

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$$u_1' = \frac{-\cancel{6\sqrt{17}}\cos x \times \cancel{e^{4x}}}{\cancel{6\sqrt{17}}\,\cancel{e^{4x}}} = -\cos x$$

$$u_2' = \frac{-\cancel{3}\cos x \times (2-\sqrt{17})e^{(2-\sqrt{17})x}}{\cancel{6}_{2}\sqrt{17}\,e^{4x}} = \frac{(\sqrt{17}-2)}{2\sqrt{17}}\cos x \times e^{(2-\sqrt{17})x}$$

$$u_3' = \frac{\cancel{3}\cos x \times (2+\sqrt{17})e^{(2+\sqrt{17})x}}{\cancel{6}_{2}\sqrt{17}\,e^{4x}} = \frac{(2+\sqrt{17})}{2\sqrt{17}}\cos x \times e^{(-2+\sqrt{17})x}$$

$$u_1 = \int -\cos x\,dx = -\sin x$$

$$u_2 = \frac{-2+\sqrt{17}}{2\sqrt{17}}\int \cos x \times e^{-(2+\sqrt{17})x}\,dx$$

*

$$\int \cos x \times e^{-(2+\sqrt{17})x}\,dx = \cos x \cdot \frac{1}{-(2+\sqrt{17})}e^{-(2+\sqrt{17})x} - \int(-\sin x)\cdot\frac{1}{-(2+\sqrt{17})}e^{-(2+\sqrt{17})x}\,dx$$

$$= \cos x \cdot \frac{e^{-(2+\sqrt{17})x}}{-(2+\sqrt{17})} - (-\sin x)\cdot\frac{1}{(2+\sqrt{17})^2}e^{-(2+\sqrt{17})x} + \int(-\cos x)\cdot\frac{e^{-(2+\sqrt{17})x}}{(2+\sqrt{17})^2}\,dx$$

$$\left(1+\frac{1}{(2+\sqrt{17})^2}\right)\int \cos x \times e^{-(2+\sqrt{17})x}\,dx = -\cos x \cdot \frac{e^{-(2+\sqrt{17})x}}{2+\sqrt{17}} + \sin x \cdot \frac{e^{-(2+\sqrt{17})x}}{(2+\sqrt{17})^2}$$

$$u_2 = \frac{\overset{3}{\cancel{-2+\sqrt{17}}}}{2\sqrt{17}}\left(\frac{\overset{2+\sqrt{17}}{\cancel{11+4\sqrt{17}}}}{12+4\sqrt{17}}\right)\left(\frac{1}{\cancel{2+\sqrt{17}}}\right)\left(-\cos x \times e^{-(2+\sqrt{17})x} + \sin x \cdot \frac{e^{-(2+\sqrt{17})x}}{2+\sqrt{17}}\right)$$

$$u_3 = \frac{2+\sqrt{7}}{2\sqrt{7}} \int \cos x \times e^{(-2+\sqrt{7})x}\, dx.$$

$$* \int \cos x \times e^{(-2+\sqrt{7})x}\, dx = \cos x \cdot \frac{e^{(-2+\sqrt{7})x}}{(-2+\sqrt{7})} - (-\sin x) \cdot \frac{e^{(-2+\sqrt{7})x}}{(-2+\sqrt{7})^2}$$

$$+ \int (-\cos x) \cdot \frac{e^{(-2+\sqrt{7})x}}{(-2+\sqrt{7})^2}\, dx.$$

$$\left(1 + \frac{1}{(-2+\sqrt{7})^2}\right) \int \cos x \times e^{(-2+\sqrt{7})x}\, dx = \cos x \cdot \frac{e^{(-2+\sqrt{7})x}}{(-2+\sqrt{7})} + \sin x \cdot \frac{e^{(-2+\sqrt{7})x}}{(-2+\sqrt{7})^2}$$

$$u_3 = \frac{\overset{3}{\cancel{2+\sqrt{7}}}}{2\sqrt{7}} \left(\frac{\cancel{(-2+\sqrt{7})^2}}{12-4\sqrt{7}}\right) \left(\frac{1}{\cancel{-2+\sqrt{7}}}\right) \left(\cos x \cdot e^{(-2+\sqrt{7})x} + \sin x \cdot \frac{e^{(-2+\sqrt{7})x}}{-2+\sqrt{7}}\right).$$

$$\therefore y_p = -\sin x + \frac{3\cancel{e^{-(2+\sqrt{7})x}}}{24\sqrt{7}+56} \left(-\cos x + \frac{\sin x}{2+\sqrt{7}}\right) \cancel{e^{(2+\sqrt{7})x}}$$

$$+ \frac{3\cancel{e^{(-2+\sqrt{7})x}}}{24\sqrt{7}-56} \left(\cos x + \frac{\sin x}{-2+\sqrt{7}}\right) \cancel{e^{(2-\sqrt{7})x}}.$$

$$= \left(-1 + \frac{3}{24\sqrt{7}+56} \cdot \frac{1}{2+\sqrt{7}} - \frac{3}{24\sqrt{7}-56} \cdot \frac{1}{-2+\sqrt{7}}\right) \sin x \quad ①$$

$$+ \left(\frac{-3}{24\sqrt{7}+56} + \frac{3}{24\sqrt{7}-56}\right) \cos x. \quad ②$$

$$(24\sqrt{7}+56)(2+\sqrt{7})$$

$$= 48\sqrt{7} + 168 + 112 + 56\sqrt{7}$$

$$= (104\sqrt{7} + 280)$$

$$(24\sqrt{7}-56)(-2+\sqrt{7}) = -48\sqrt{7} + 168 + 112 - 56\sqrt{7}$$

$$= -104\sqrt{7} + 280$$

①

$$-1 + \frac{3(-104\sqrt{7}+280 - 104\sqrt{7} - 280)}{(104\sqrt{7}+280)(-104\sqrt{7}+280)}$$

$$= -1 + \frac{3(-208\sqrt{7})}{2688} = -1 - \frac{208}{896}\sqrt{7} \quad ?$$

②

$$3\left(\frac{\cancel{24\sqrt{7}}+56-\cancel{24\sqrt{7}}+56}{896}\right) = \frac{3}{8}$$