

2.5 문제 5.1 (5)

Step 1)

$$\lambda^2 - 1 = 0 \quad \lambda = -1, 1$$

$$\Rightarrow y_h = c_1 e^{-x} + c_2 e^x$$

Step 2) $(D^2 - 1)y_p = e^x.$

$$(D-1)y_p = \frac{1}{D+1} e^x$$

$$= \frac{1}{2} e^x.$$

$$y_p = \frac{1}{D-1} \left(\frac{1}{2} e^x \right)$$

$$= e^x \frac{1}{(D+1)-1} \left(\frac{1}{2} \right)$$

$$= e^x \frac{1}{D} \left(\frac{1}{2} \right) = \underline{\underline{\frac{x}{2} e^x}}$$

$$\therefore y = y_h + y_p = c_1 e^{-x} + c_2 e^x + \frac{x}{2} e^x$$

↑ 방법 1.

Step 2). $(D-1)y_p = \frac{1}{D+1} e^x = e^{-x} \int e^x \cdot e^x dx = e^{-x} \int e^{2x} dx.$

$$= e^{-x} \left(\frac{1}{2} e^{2x} \right) = \frac{1}{2} e^x$$

$$y_p = \frac{1}{D-1} \left(\frac{1}{2} e^x \right) = e^x \int \frac{1}{2} e^{-x} \cdot e^x dx = \underline{\underline{\frac{1}{2} x e^x}}$$

$$\text{답지: } y_p = \left(\frac{x}{2} - \frac{1}{4} \right) e^x$$

↑ 방법 2.