

2.3/ 01/21/ 8.11

$$4m^2 + 4m + 5 = 0$$

$$m = \frac{-2 \pm \sqrt{1}}{4} = -\frac{1}{2} \pm \frac{1}{4}i$$

$$\therefore \text{일반해: } y = x^{\frac{1}{2}} \left\{ C_1 \cos\left(\frac{1}{4} \ln|x|\right) + C_2 \sin\left(\frac{1}{4} \ln|x|\right) \right\}$$

$$y(1) = e^{\pi}$$

$$\rightarrow e^{\pi} = C_1$$

$$y(e^{\frac{\pi}{8}}) = e^{\frac{3\pi}{8}}$$

$$\rightarrow e^{\frac{3\pi}{8}} = e^{\frac{\pi}{8}} \left\{ e^{\pi} \cos\left(\frac{\pi}{8}\right) + C_2 \sin\left(\frac{\pi}{8}\right) \right\}$$

$$\rightarrow C_2 = \frac{e^{\pi} (1 - \cos(\frac{\pi}{8}))}{\sin(\frac{\pi}{8})} = \frac{2e^{\pi}}{\sin(\frac{\pi}{8})} \cdot \frac{1 - \cos\frac{\pi}{8}}{2} = \frac{2e^{\pi}}{\sin(\frac{\pi}{8})} \cdot \sin\left(\frac{\pi}{16}\right)$$

$$= \frac{2e^{\pi} \sin\frac{\pi}{16}}{\cancel{2\sin\frac{\pi}{8}} \cos\frac{\pi}{16}} = \boxed{e^{\pi} \sec\frac{\pi}{16}}$$

$$y(e^{\frac{\pi}{8}}) = e^{\frac{\pi}{8}} \left\{ e^{\pi} \cos\left(\frac{\pi}{16}\right) + e^{\pi} \sec\left(\frac{\pi}{16}\right) \right\}$$

$$= e^{\frac{9\pi}{8}} \left(\cos\frac{\pi}{16} + \sec\frac{\pi}{16} \right)$$

$$\cos\frac{2\pi}{16} = \frac{1 + \cos\frac{\pi}{8}}{2} \quad \cos\frac{2\pi}{8} = \frac{1 + \frac{\sqrt{2}}{2}}{2} ?$$