2.1 / 예제 1.2

$y''y^3 = -1$

$y' = u$, $y'' = u\frac{du}{dy}$

$u \cdot \frac{du}{dy} \cdot y^3 = -1$

$\int u\,du = \int -\frac{1}{y^3}\,dy$

$\left[\frac{1}{2}u^2 = \frac{1}{2}\cdot\frac{1}{y^2} + C_1\right] \times 2$

$u^2 = \frac{1}{y^2} + C_1$

$u = \pm\sqrt{\frac{1+C_1y^2}{y^2}} = \frac{dy}{dx}$

$\int dx = \int \frac{\pm y}{\sqrt{1+C_1y^2}}\,dy = \pm\frac{1}{2C_1}\int \frac{2C_1y}{\sqrt{1+C_1y^2}}\,dy$

$x + C_2 = \pm\frac{1}{2C_1} \times 2\sqrt{1+C_1y^2}$

$(x+C_2)^2 = \frac{1+C_1y^2}{C_1^2}$ (*) $y^2 = C_1(x+C_2)^2 - \frac{1}{C_1}$

$\begin{cases} f(0)=1 \;\; (*): 1 = C_1C_2^2 = \frac{1}{C_1} \cdots ① \\ f'(0)=-1. \;\; (*) \to \text{양변} \to 2y\cdot y' = 2C_1(x+C_2) \end{cases}$

대입 $\to -2 = 2C_1C_2$.

$\therefore C_1C_2 = -1 \cdots ②$, ②를 ①에 대입.

① : $C_1 = C_1^2C_2^2 - 1 = -2$, $C_2 = \frac{1}{2}$

$\therefore x = -b$ 일때 (*) : $y^2 = -2(-b+\frac{1}{2})^2 + \frac{1}{2} < 0$ … ?