A piston-cylinder : air & liquid water $\longrightarrow$ compression process
(State 1 $\longrightarrow$ State 2)

Stage 1 : $T_1 = 25^\circ C$ $p_1 = 100\, kPa$ the air contains no water vapor the cylinder volume = $1\, m^3$ mass of liq. water : 1 kg	Stage 2 : a mixture of water vapor and air $T_2 = 180^\circ C$ $V_2 = 0.1\, m^3$

The volume occupied by the liquid can be neglected
Air is assumed to be an ideal gas with $C_v = 0.728\, kJ/kg \cdot K$, $R = 0.287\, kJ/kg \cdot K$

(a) Assuming : the water vapor and air is an ideal mixture.
Determine the pressure inside the cylinder at State 2 , p_2.
Neglect any air dissolved in the water

(b) During the compression : $pV^n = \text{constant}$
Determine the total amount of work during the compression

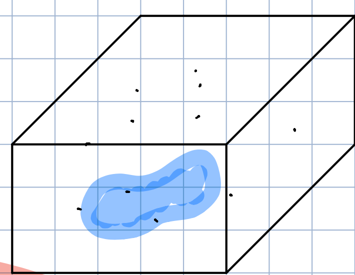
(c) Determine the heat transfer from the surrounding into the piston-cylinder process

(a) Assuming : the water vapor and air is an ideal mixture.
Determine the pressure inside the cylinder at State 2 , p_2.
Neglect any air dissolved in the water

$$p_2 = p_{2,air} + p_{2,wv}$$

· state 2 : $p_2 V_2 = mRT_2$

$$p_{2,air} = \frac{mRT_2}{V_2} \longleftarrow \quad T_2 = 180^\circ C \quad V_2 = 0.1\, m^3 \quad m = ? \qquad R = 0.287\, kJ/kg \cdot K$$



· state 1 : Table A-4 $T_1 = 25^\circ C \rightarrow p_{sat} = 3.17\, kPa$

$$p_1 = \underbrace{p_{1,wv}}_{= 3.17\, kPa} + p_{1,air} = 100\, kPa \qquad \therefore p_{1,air} = 96.83\, kPa$$

$$p_1 V_1 = mRT_1 \longrightarrow m_{air} = \frac{p_1 V_1}{RT_1}$$ (∵ volume occupied by liquid : neglected)

$$= \frac{(96.83\, kPa) \cdot (1\, m^3)}{(0.287\, kJ/kg \cdot K) \cdot (298\, K)} = 1.13\, kg$$

$$\therefore p_{2,air} = \frac{(1.13\, kg) \cdot (0.287\, kJ/kg \cdot K) \cdot (453\, K)}{(0.1\, m^3)} = 1469.1\, kPa$$

· state 2 : $T_2 = 180^\circ C$ Table A-4 $\longrightarrow p_{2,wv} = 1002.8\, kPa$

$$\therefore p_2 = p_{2,air} + p_{2,wv} = 2471.9\, kPa$$ Ans.

(b) During the compression : $pV^n = \text{constant}$

Determine the total amount of work during the compression

$$W = \int_{V_1}^{V_2} p\,dV \longleftarrow \boxed{pV^n = \text{const}} = p_1V_1^{\,n} = p_2V_2^{\,n}, \quad p = \text{const}/V^n = \left(p_1V_1^{\,n}/V^n\right)$$

$$= \int_{V_1}^{V_2} \frac{\text{const}}{V^n}\,dV$$

$$= \text{const} \cdot \frac{1}{1-n} \cdot V^{1-n}\Big]_{V_1}^{V_2} \qquad 1 < n < k$$

$$= \frac{\text{const}}{1-n} \cdot \left(V_2^{\,1-n} - V_1^{\,1-n}\right)$$

$$= \frac{p_2V_2^{\,n} \cdot V_2^{\,1-n} - p_1V_1^{\,n} \cdot V_1^{\,1-n}}{1-n} = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{(2471.9)(0.1) - (100)(1)}{1-1.393} = -374.53\text{ kJ}$$

Ans.

$n = ?$

$$p_1V_1^{\,n} = p_2V_2^{\,n}$$

$$(100)(1)^n = (2471.9) \cdot (0.1)^n$$

$$100/2471.9 = 0.1^n$$

$$n \log(0.1) = \log(100/2471.9) \longrightarrow n = 1.393$$

(C) Determine the heat transfer from the surrounding into the piston-cylinder process

State 1	→	State 2
Liq. water air		Liq. water + water vapor air

$$\Delta E = Q - W = \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$Q - (-W_{in} + W_b) = \Delta U$$

$$Q = \Delta U + W_b - W_{in} \qquad H = U + PV$$

$$\Delta H = \Delta U + \Delta(PV)$$

$$Q = \Delta H - W_{in} \longleftarrow dH = dU + \cancel{dP \cdot V} + P \cdot dV$$

$= \delta W_b$

$$\boxed{Q = \Delta H - W_{in}}$$

$$= (\Delta H)_{air} + (\Delta H)_{wv} + (\Delta H)_{wl} - W_{in}$$

$$= \{mC_p(T_2 - T_1)\}_{air} + \{m_2h_2 - m_1h_1\}_{wv} + \{m_2h_2 - m_1h_1\}_{wl} - W_{in}$$

- $(\Delta H)_{air} = m_{air}C_p \cdot (T_2 - T_1) \longleftarrow C_p - C_v = R$

$$C_p = C_v + R$$

$$= (1.13\text{ kg})(1.015\text{ kJ/kg}\cdot\text{K}) \cdot (180 - 25)\text{ K}$$

$$= 177.78\text{ kJ}$$

m_1, liq. water = 1kg

if all liq. water turned into sat. vapor $v_2 = 0.1\ m^3/kg$

Table A-4 T=180°C ⟶ $v_f = 0.001127$ ~ $v_g = 0.19384$

$$m_{2,wv} = V_2/v_g = \frac{0.1\ m^3}{0.19384\ m^3/kg} = 0.51589\ kg$$

$$m_{2,wl} = 1 - 0.51589 = 0.484\ kg$$

$$\cdot(\Delta H)_{wv} = m_{2,wv} \cdot h_{2,wv} - \cancel{m_{1,wv} \cdot h_{1,wv}}$$

no water vapor at stage 1

$$= (0.5159\ kg) \cdot (1802.16) = 929.73\ kJ$$

T=180°C, Table A-4 ⟶ $h_f = 763.05$
$h_{fg} = 2014.2$
$h_g = 2777.2$

$$x_2 = \frac{0.5159}{1} \longrightarrow h_{2,wv} = 1802.16$$

$$(\Delta H)_{wl} = m_{2,wl} \cdot h_{2,wl} - m_{1,wl} \cdot h_{1,wl}$$

* $m_{2,wl} = 0.484\ kg$ $h_{2,wl} = h_f = 763.05$

$m_{1,wl} = 1kg$ $h_{1,wl} = 104.83$ ← Table A-4, $T_i = 25°C$

$$= (0.484) \cdot (763.05) - (1)(104.83) = 264.48$$

$$\therefore Q = (177.78) + (929.73) + (264.48) - 374.53$$

$$= 997.46\ kJ$$ Ans.