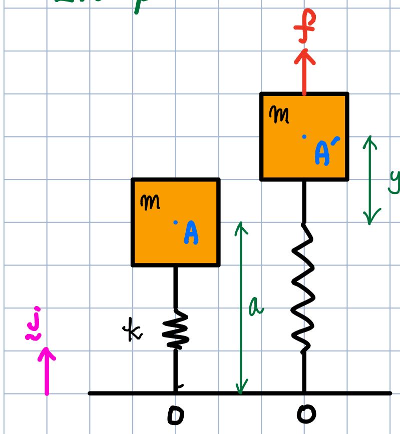
Example >



Step1. The position vector

$$\underset{\sim}{r}_{A'} = (a+y)\,\underset{\sim}{j} \longrightarrow \dot{\underset{\sim}{r}}_{A'} = \dot{y}\,\underset{\sim}{j}$$

Step2. The kinetic energy

$$T = \frac{1}{2}m(\dot{\underset{\sim}{r}}_{A'}\cdot\dot{\underset{\sim}{r}}_{A'}) = \frac{1}{2}m\dot{y}^2$$

Step3. The position energy

$$V = V_{spring} + V_{gravity}$$

$$V_{spring} = \int_y^0 (-ky\,\underset{\sim}{j})\cdot d\underset{\sim}{r}_{A'} = \int_y^0 -ky\,dy = \frac{1}{2}ky^2$$

$= d(a+y)\underset{\sim}{j}$

$= da + dy\,\underset{\sim}{j}$

$$V_{gravity} = \int_y^0 -mg\,dy = mgy$$

Step4 Nonconservative work

$$\delta W_{noncons} = f\,\underset{\sim}{j}\cdot\delta\underset{\sim}{r}_{A'} = f\cdot\delta(a+y) = f\cdot\delta y$$

$$Q_y = f$$

Step5. Euler-Lagrange's Equation of Motion

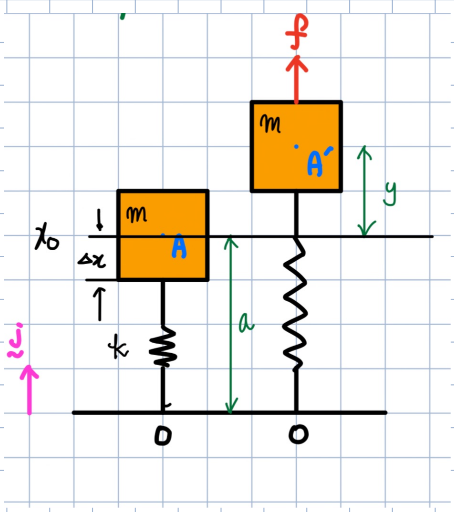
$$\boxed{\underset{\sim}{Q}_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}}$$

$$L = T - V = \frac{1}{2}m\dot{y}^2 - \frac{1}{2}ky^2 - mgy$$

$$\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial \dot{y}} = m\dot{y} \qquad \frac{\partial L}{\partial q_i} = \frac{\partial L}{\partial y} = -ky - mg$$

$$m\ddot{y} + ky + mg = f$$

Ans.



1. Position Vector

$$\vec{r}_{A'} = (a+y)\vec{i}$$

$$\dot{\vec{r}}_{A'} = \dot{y}\vec{i}$$

2. Kinetic energy

$$T = \frac{1}{2}m\dot{r}_{A'}^2 = \frac{1}{2}m\dot{y}^2$$

3. Potential energy

$$V_{spring} = \int_y^0 -k\cdot(y-\Delta x)\,dy = \frac{1}{2}ky^2 - k\Delta x y$$

$$V_{gravity} = \int_y^0 -mg\cdot dy = mgy$$

$$\therefore L = T - U = \frac{1}{2}m\dot{y}^2 - \frac{1}{2}ky^2 + k\Delta x y - mgy$$

$$\frac{\partial L}{\partial \dot{y}} = m\dot{y} \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m\ddot{y}$$

$$\frac{\partial L}{\partial y} = -ky + k\Delta x - mg \longleftarrow kx - mg = 0 \text{ for force balance}$$

$$\boxed{Q_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}} \longrightarrow m\ddot{y} + ky = f$$

Ans

We don't have to consider gravity terms if the system started from static position