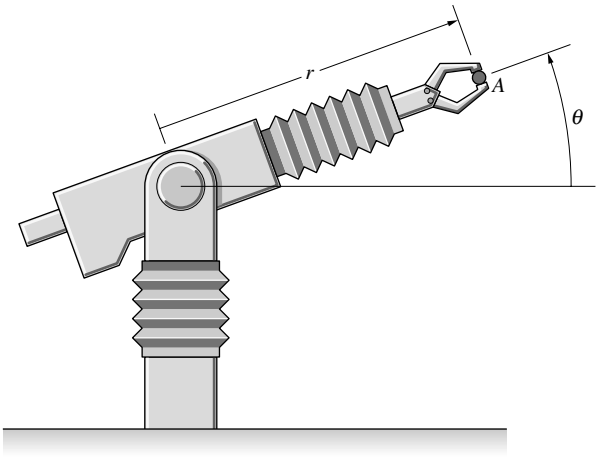
Problem 14.96 The robot is programmed so that the 0.4-kg part A describes the path

$r = 1 - 0.5\cos(2\pi t)$ m,

$\theta = 0.5 - 0.2\sin(2\pi t)$ rad.

Determine the radial and transverse components of the force exerted on A by the robot's jaws at $t = 2$ s.



Solution: The radial component of the acceleration is

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2.$$

The derivatives:

$$\frac{dr}{dt} = \frac{d}{dt}(1 - 0.5\cos 2\pi t) = \pi \sin 2\pi t,$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}(\pi \sin 2\pi t) = 2\pi^2 \cos 2\pi t;$$

$$\frac{d\theta}{dt} = \frac{d}{dt}(0.5 - 0.2\sin 2\pi t) = -0.4\pi \cos 2\pi t.$$

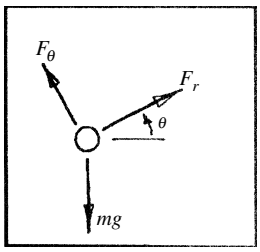
$$\frac{d^2\theta}{dt^2} = \frac{d}{dt}(-0.4\pi \cos 2\pi t) = 0.8\pi^2 \sin 2\pi t.$$

From which

$$[a_r]_{t=2} = 2\pi^2 \cos 4\pi - (1 - 0.5\cos 4\pi)(-0.4\pi \cos 4\pi)^2,$$

$$= 2\pi^2 - 0.08\pi^2 = 18.95 \text{ m/s}^2;$$

$\theta(t = 2) = 0.5$ rad.



From Newton's second law, $F_r - mg\sin\theta = ma_r$, and $F_\theta - mg\cos\theta = ma_\theta$, from which

$$\boxed{F_r = 0.4a_r + 0.4g\sin\theta = 9.46 \text{ N}.}$$

The transverse component of the acceleration is

$$a_\theta = r\left(\frac{d^2\theta}{dt^2}\right) + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right),$$

from which $[a_\theta]_{t=2} = (1 - 0.5\cos 4\pi)(0.8\pi^2 \sin 4\pi) + 2(\pi \sin 4\pi)(-0.4\pi \sin 4\pi) = 0$, and

$$\boxed{F_\theta = 3.44 \text{ N}}$$