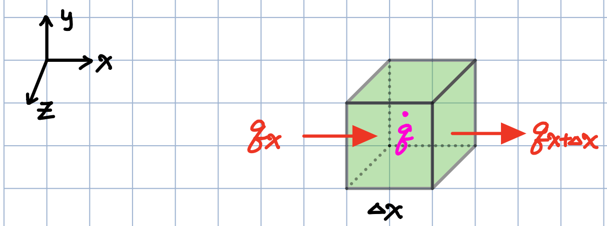
Rectangular Coordinate with Heat Generation



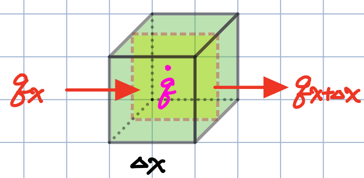
$$\dot{E}_{st} = \dot{E}_g + \dot{E}_{in} - \dot{E}_{out}$$

$$\rho(dxdydz)\cdot c\cdot \cancel{\frac{\partial T}{\partial t}} = \dot{q}(dxdydz) + \cancel{q_x} - \left(\cancel{q_x} + \frac{\partial q_x}{\partial x}\cdot dx\right) \leftarrow q_x = -kA\frac{dT}{dx} = -k(dydz)\frac{dT}{dx}$$

steady-state

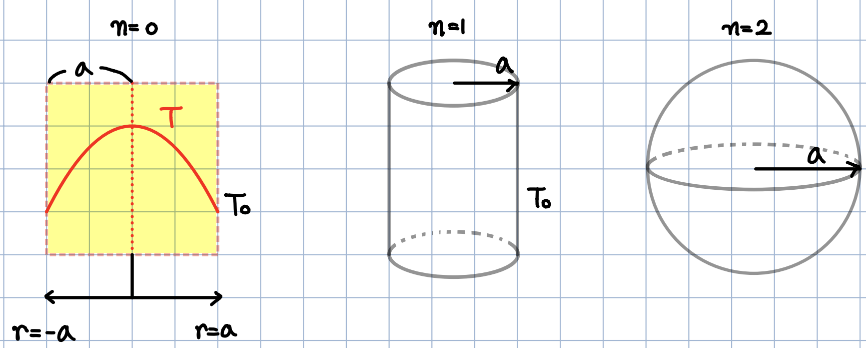
$$0 = \dot{q}\cancel{(dxdydz)} - \frac{d}{dx}\left\{-k\cancel{(dydz)}\frac{dT}{dx}\right\}\cdot\cancel{dx}$$

$$\boxed{-k\nabla^2 T = \dot{q}}$$



$\dot{q}$ = constant

$$\nabla^2 = \frac{1}{r^n}\cdot\frac{d}{dr}\cdot r^n\cdot\left(\frac{d}{dr}\right)$$



$$-k\cdot\left\{\frac{1}{r^n}\cdot\frac{d}{dr}\cdot r^n\cdot\left(\frac{dT}{dr}\right)\right\} = \dot{q}$$

$$\int -k\cdot\left\{r^n\cdot d\left(\frac{dT}{dr}\right)\right\} = \int \dot{q}\, r^n dr$$

$$-k\cdot r^n\cdot\left(\frac{dT}{dr}\right) = \dot{q}\cdot\frac{1}{n+1}r^{n+1} + \cancel{C_1}$$

symmetric: $\left.\frac{dT}{dr}\right|_{r=0} = 0$, $C_1 = 0$

$$\int -k\cdot dT = \int \frac{\dot{q}}{n+1}\cdot r\, dr$$

$$-kT = \frac{\dot{q}r^2}{2(n+1)} + C_2$$

$$T = -\frac{\dot{q}r^2}{2(n+1)k} + C_3 \leftarrow r=a,\ T=T_0$$

$$T_0 = -\frac{\dot{q}a^2}{2(n+1)k} + C_3$$

$$\therefore C_3 = T_0 + \frac{\dot{q}a^2}{2(n+1)k}$$

$$\therefore T = -\frac{(r^2-a^2)\dot{q}}{2(n+1)k} + T_0 \longrightarrow \boxed{T - T_0 = \frac{\dot{q}a^2}{2(n+1)k}\left(1 - r^2/a^2\right)}$$

slab $(n=0)$: $T - T_0 = \frac{\dot{q}a^2}{2k}\left(1 - x^2/a^2\right)$

cylinder $(n=1)$: $T - T_0 = \frac{\dot{q}a^2}{4k}\left(1 - r^2/a^2\right)$

sphere $(n=2)$: $T - T_0 = \frac{\dot{q}a^2}{6k}\left(1 - r^2/a^2\right)$