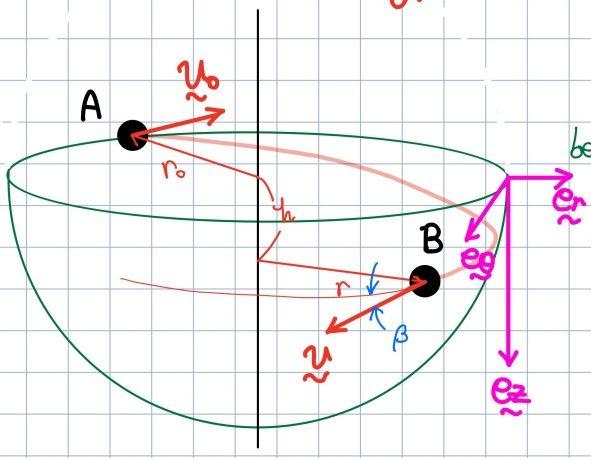
Example > 3.27 (Energy)



A small mass particle is given an initial velocity v_0 at r_0.

As the particle slides past point B, a distance h below A and a distance r from the vertical centerline, its velocity v makes angle β with the horizontal tangent to the bowl through B.

Determine β.

* $(H_O)_1 = (H_O)_2$ conservation of angular momentum

$$\underset{\sim}{H_O} = \underset{\sim}{r} \times m\underset{\sim}{v}$$

$$= rmv\sin 90^\circ$$

$$= rmv$$

$(H_O)_1 = r_0 m v_0$ $\longrightarrow$ $mv_0 r_0 = mvr\cos\beta$ $\therefore$ $v_0 r_0 = \sqrt{v_0^2 + 2gh} \cdot \sqrt{r_0^2 - h^2} \cdot \cos\beta$

$(H_O)_2 = rmv\cos\beta$

* $E_1 = E_2$ conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv^2 + 0$$

$$\therefore v = \sqrt{v_0^2 + 2gh}$$

* $r^2 = r_0^2 - h^2$

$$\therefore \beta = \cos^{-1}\left(\frac{1}{\sqrt{1+\left(\frac{2gh}{v_0}\right)} \cdot \sqrt{1-\left(\frac{h}{r_0}\right)^2}}\right)$$

Ans.