A piston-cylinder : air & liquid water $\longrightarrow$ compression process

(State 1 $\rightarrow$ State 2)

<table>
<tr><td>Stage 1 :</td><td>$T_1 = 25°C$
$p_1 = 100$ kPa
the air contains no water vapor
the cylinder volume = 1 m^3
mass of liq. water : 1 kg</td><td>Stage 2 :</td><td>a mixture of water vapor and air
$T_2 = 180°C$
$V_2 = 0.1\ m^3$</td></tr>
</table>

The volume occupied by the liquid can be neglected

Air is assumed to be an ideal gas with $c_v = 0.718$ kJ/kg·K , $R = 0.287$ kJ/kg·K

(a) Assuming : the water vapor and air is an ideal mixture.
Determine the pressure inside the cylinder at State 2 , p_2.
Neglect any air dissolved in the water

(b) During the compression : $pV^n = $ constant
Determine the total amount of work during the compression

(c) Determine the heat transfer from the surrounding into the piston-cylinder process

(a) Assuming : the water vapor and air is an ideal mixture.
Determine the pressure inside the cylinder at State 2 , p_2.
Neglect any air dissolved in the water

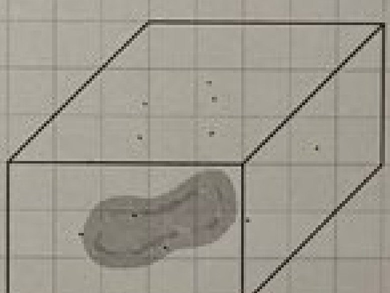
$p_2 = p_{2,air} + p_{2,wv}$

· state2 : $p_2 V_2 = mRT_2$

$$p_{2,air} = \frac{mRT_2}{V_2} \longleftarrow \quad T_2 = 180°C \quad R = 0.287 \text{ kJ/kg·K}$$

$V_2 = 0.1\ m^3$

$m = ?$



· state1 : Table A-4 $T_1 = 25°C \rightarrow P_{sat} = 3.17$ kPa

$p_1 = p_{1,wv} + p_{1,air} = 100$ kPa $\quad \therefore p_{1,air} = 96.83$ kPa

$= 3.17$ kPa

$p_1 V_1 = mRT_1 \longrightarrow m_{air} = \dfrac{p_1 V_1}{RT_1}$ (∵ volume occupied by liquid : neglected)

$$= \frac{(96.83\ \text{kPa}) \cdot (1\ m^3)}{(0.287\ \text{kJ/kg·K}) \cdot (298\ \text{K})} = 1.13\ \text{kg}$$

$$\therefore p_{2,air} = \frac{(1.13\ \text{kg}) \cdot (0.287\ \text{kJ/kg·K}) \cdot (453\ \text{K})}{(0.1\ m^3)} = 1469.1\ \text{kPa}$$

· state2 : $T_2 = 180°C$ Table A-4 $\longrightarrow p_{2,wv} = 1002.8$ kPa $\quad \therefore p_2 = p_{2,air} + p_{2,wv}$

$= 2471.9$ kPa Ans

(b) During the compression : pV^n = constant

Determine the total amount of work during the compression

$$W = \int_{V_1}^{V_2} p\,dV \quad \longleftarrow \quad pV^n = \text{const} = p_1V_1^n = p_2V_2^n, \quad p = \text{const}/V^n = p_1V_1^n/V^n$$

$$= \int_{V_1}^{V_2} \frac{\text{const}}{V^n}\,dV$$

$$= \text{const} \cdot \frac{1}{1-n} \cdot V^{1-n} \Big]_{V_1}^{V_2} \qquad 1 < n < k$$

$$= \frac{\text{const}}{1-n} \cdot \left(V_2^{1-n} - V_1^{1-n}\right)$$

$$= \frac{p_2V_2^n \cdot V_2^{1-n} - p_1V_1^n \cdot V_1^{1-n}}{1-n} = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{(2471.9)(0.1) - (100)(1)}{1-1.393} = -374.53\ \text{kJ}$$

Ans.

$n = ?$

$$p_1V_1^n = p_2V_2^n$$

$$(100)(1)^n = (2471.9) \cdot (0.1)^n$$

$$100/2471.9 = 0.1^n$$

$$n \log(0.1) = \log(100/2471.9) \longrightarrow n = 1.393$$

(c) Determine the heat transfer from the surrounding into the piston-cylinder process

State 1		State 2
Liq. water air	$\longrightarrow$	Liq. water + water vapor air

$$\Delta E = Q - W = \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$Q - (-W_{in} + W_b) = \Delta U$$

$$Q = \Delta U + W_b - W_{in} \qquad H = U + PV$$

$$\Delta H = \Delta U + \Delta(PV)$$

$$Q = \Delta H - W_{in} \quad \longleftarrow \quad dH = dU + \cancel{dP\,V} + \underbrace{P\,dV}_{=\delta W_b}$$

$$Q = \Delta H - W_{in}$$

$$= (\Delta H)_{air} + (\Delta H)_{wv} + (\Delta H)_{wl} - W_{in}$$

$$= \{mC_p(T_2 - T_1)\}_{air} + \{m_2h_2 - m_1h_1\}_{wv} + \{m_2h_2 - m_1h_1\}_{wl} - W_{in}$$

$$\cdot\ (\Delta H)_{air} = m_{air}C_p \cdot (T_2 - T_1) \quad \longleftarrow \quad C_p - C_v = R$$

$$C_p = C_v + R$$

$$= (1.13\ \text{kg})(1.015\ \text{kJ/kg}\cdot\text{K}) \cdot (180 - 25)\ \text{K}$$

$$= 177.78\ \text{kJ}$$

질문)

밀폐계에서 1법칙은

$$\Delta E = Q - W$$

이고, 교수님의 말대로 $W = -W_{in} + W_b$ 으로 나누어질 수 있으며,

여기서 $W_b = \int p\,dV$ 라는 것까지 이해했습니다.

그런데,

질문 1)

(b)에서 compression 중 총 일을 $\int p\,d\forall$ 식을 이용해 구했습니다.

그렇다면 (b)에서는 W_b를 구한 것이므로,

$$Q = \Delta U + W_b - W_{in}$$

이 식에서 W_b를 구한 것 아닌가요? 교수님은 (b)에서 구한 일이 W_{in}이라고

하셔서 의아합니다.

질문 2)

만약 (b)에서 구한 일이 시스템이 한 일이 아니라 외부에서 해준 일이므로

W_{in}으로 들어가야 한다면, W_b와 W_{in}은 어떻게 구분하나요?

$\delta W_b = p \cdot dV$ 식에 따라 W_b를 구하고, 문제에서 제시한 W_{in}은

따로 넣어 주는 방식으로 문제를 풀어 왔는데, 이 문제에서는 W_{in}이

제시되어 있지 않아 구분할 수 없는 것 같아 질문드립니다.