#5.21.

(가) $\tanh z = \dfrac{e^{z}-e^{-z}}{e^{z}+e^{-z}} = \dfrac{e^{2z}-1}{e^{2z}+1}$ ∴ (가) (o).

$\cosh z = \dfrac{e^{z}+e^{-z}}{2}$

$e^{2\pi i} = \cos 2\pi + i \cdot \sin 2\pi = 1$

$e^{\pi i} = \cos \pi + i \cdot \sin \pi = -1$

(나) $\sin z = \dfrac{e^{iz}-e^{-iz}}{2i}$, $|\sin z|^2 = \dfrac{-(e^{2iz}+e^{-2iz}-2)}{+4}$

$\cos z = \dfrac{e^{iz}+e^{-iz}}{2}$ $|\cos z|^2 = \dfrac{e^{2iz}+e^{-2iz}+2}{4}$ (나) (o).

(다)

$w = \tan^{-1} z$. $z = \tan w$.

$z = \dfrac{e^{iw}-e^{-iw}}{i(e^{iw}+e^{-iw})} = \dfrac{e^{2iw}-1}{i(e^{2iw}+1)}$

$i(e^{2iw}+1)z = e^{2iw}-1$ $e^{2iw} \times (iz-1) = -iz-1$

$e^{2iw} = \dfrac{-iz-1}{iz-1}$ $2iw = \ln\left(\dfrac{z-i}{-z-i}\right) = \ln\left(\dfrac{i-z}{i+z}\right)$

$w = \dfrac{1}{2i} \ln\left(\dfrac{1+iz}{1-iz}\right) = -\dfrac{i}{2} \ln\left(\dfrac{1-iz}{1+iz}\right)$ (다) (x).

(라) ?

$z = x + iy. \quad |\mathrm{Im}(z)| = |y| \geq 1.$

$$|\tan z| \leq \left(1 + \frac{1}{\sinh^2 1}\right)^{\frac{1}{2}} \qquad \sinh 1 = \frac{e^{1} - e^{-1}}{2}$$

$$= \left(1 + \left(\frac{2}{e - e^{-1}}\right)^2\right)^{\frac{1}{2}} = \left(\frac{e^{2} + e^{-2} - \cancel{2} + \cancel{4}^{\,2}}{(e - e^{-1})^2}\right)^{\frac{1}{2}} = \frac{e + e^{-1}}{e - e^{-1}}$$

$$\tan z = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}} = \frac{e^{2z} - 1}{e^{2z} + 1}$$

$$= \frac{e^{2x + i \cdot 2y} - 1}{e^{2x + i \cdot 2y} + 1} = \frac{e^{2x} \cdot (\cos 2y + i \cdot \sin 2y) - 1}{e^{2x} \cdot (\cos 2y + i \cdot \sin 2y) + 1}$$

$$= \frac{(e^{2x} \cdot \cos 2y - 1) + i \cdot e^{2x} \cdot \sin 2y}{(e^{2x} \cdot \cos 2y + 1) + i \cdot e^{2x} \cdot \sin 2y}$$

$$= \frac{\left((e^{2x} \cdot \cos 2y - 1) + i \cdot e^{2x} \cdot \sin 2y\right) \times \left((e^{2x} \cdot \cos 2y + 1) - i \cdot e^{2x} \cdot \sin 2y\right)}{(e^{2x} \cdot \cos 2y + 1)^2 + (e^{2x} \cdot \sin 2y)^2}$$

$$= \frac{\overset{e^{4x}}{\cancel{(e^{2x} \cdot \cos 2y)^2}} - 1 + \cancel{(e^{2x} \cdot \sin 2y)^2} + 2i \cdot e^{2x} \cdot \sin 2y}{\overset{e^{4x}}{\cancel{(e^{2x} \cdot \cos 2y)^2 + (e^{2x} \cdot \sin 2y)^2}} + 2e^{2x} \cdot \cos 2y + 1}$$

$$= \frac{(e^{4x} - 1) + i \cdot 2e^{2x} \cdot \sin 2y}{e^{4x} + 2e^{2x} \cdot \cos 2y + 1}, \qquad |\tan z| = \frac{\sqrt{(e^{4x} - 1)^2 + 4e^{4x} \cdot \sin^2 2y}}{e^{4x} + 1 + 2e^{2x} \cdot \cos 2y}$$

(마).

$e^{e^{z}} = 1. \quad e^{z} = \ln(1) = 2k\pi \cdot i$

$\ln(1) = \cancel{\ln|1|}^{\,0} + i(2k\pi) = 2k\pi \cdot i$

$z = \ln(2k\pi i)$

$= \ln|2k\pi| + i(0 + 2n\pi)$

$= \ln|2k\pi| + 2n\pi \cdot i$

(마) (X).

(나). $z = x+iy$

$\sin z = \frac{e^{iz} - e^{-iz}}{2i} = \frac{e^{-y+ix} - e^{y-ix}}{2i} = \frac{\left(e^{-y}(\cos x + i\sin x) - e^{y}(\cos x - i\sin x)\right) \times i}{\cancel{2i}\ -2}$

$= \frac{e^{y}(\cos x - i\sin x)i - e^{-y}(\cos x + i\sin x)i}{2} = \frac{(e^{y}\sin x + e^{-y}\sin x) + i(e^{y}\cos x - e^{-y}\cos x)}{2}$

$|\sin z|^2 = \frac{1}{4} \times \left(\sin^2 x \cdot (e^{y}+e^{-y})^2 + \cos^2 x (e^{y} - e^{-y})^2\right)$

$= \frac{1}{4} \times \left(e^{2y} + e^{-2y} + 2\sin^2 x - 2\cos^2 x\right)$

$\cos z = \frac{e^{iz} + e^{-iz}}{2} = \frac{e^{-y+ix} + e^{y-ix}}{2} = \frac{e^{-y} \times (\cos x + i\sin x) + e^{y}(\cos x - i\sin x)}{2}$

$|\cos z|^2 = \frac{1}{4} \times \left(\cos^2 x (e^{y}+e^{-y})^2 + \sin^2 x \times (e^{y} - e^{-y})^2\right) = \frac{e^{2y} + e^{-2y} + 2\cos^2 x - 2\sin^2 x}{4}$

$|\cos z|^2 + |\sin z|^2 = \frac{e^{2y} + e^{-2y}}{2}$ (나) (X).