Example 2.1.1

Compute and plot the response of a spring–mass system modeled by equation (2.2) to a force of magnitude 23 N, driving frequency of twice the natural frequency, and initial conditions given by $x_0 = 0$ m and $v_0 = 0.2$ m/s. The mass of the system is 10 kg and the spring stiffness is 1000 N/m.

$F_0 = 23N \quad W = 2 \cdot W_n, \quad m = 10kg, \quad k = 1000\,N/m, \quad W_n = 10, \quad W = 20$

$x_0 = 0, \quad V_0 = 0.2$

$\underline{m\ddot{x} + kx = F_0 \cos wt}$ Equation of Motion $\xrightarrow{\div m}$ $\ddot{x} + W_n^2 x = f_0 \cos wt, \quad f_0 = F_0/m$

Let $X_p = X \cdot \cos wt$

$\dot{X}_p = -W \cdot X \cdot \sin wt$

$\ddot{X}_p = -W^2 \cdot X \cdot \cos wt$

EOM $\longrightarrow$ $-W^2 X \cos wt + W_n^2 (X \cdot \cos wt) = f_0 \cos wt$

$X \cdot \cos wt (W_n^2 - W^2) = f_0 \cos wt \qquad X = \dfrac{f_0}{W_n^2 - W^2}$

$X_p = \dfrac{f_0}{W_n^2 - W^2} \cdot \cos wt$

1) homogenous solution $X_h = A \cdot \sin(W_n t + \varnothing)$ 라 할 경우.

$X = X_h + X_p$

$= A \cdot \sin(W_n t + \varnothing) + X_p$.

$A = \dfrac{\sqrt{V_0^2 + x_0^2 W_n^2}}{W_n} = \sqrt{x_0^2 + V_0^2 / W_n^2}, \quad \varnothing = \tan^{-1}\left(\dfrac{x_0 W_n}{V_0}\right)$

$X = \sqrt{X_0^2 + V_0^2/W_n^2} \cdot \sin\left[W_n t + \tan^{-1}\left(\dfrac{x_0 W_n}{V_0}\right)\right] + \dfrac{f_0}{W_n^2 - W^2} \cos wt$

$= \sqrt{0^2 + 0.2^2/10^2} \cdot \sin\left[10t + \tan^{-1}\left(\dfrac{0 \cdot 10}{0.2}\right)\right] + \dfrac{2.3}{10^2 - 20^2} \cdot \cos 20t$

$= 0.02 \cdot \sin(10t) - 7.667 \times 10^{-3} \cdot \cos 20t$

2) homogenous solution $X_h = A_1 \sin W_n t + A_2 \cos W_n t$ 라 할 경우

$X = X_h + X_p$

$= A_1 \sin W_n t + A_2 \cos W_n t + \dfrac{f_0}{W_n^2 - W^2} \cdot \cos wt$

EOM $= \ddot{x} + W_n^2 \cdot x = f_0 \cdot \cos wt$

$X(0) = X_0 = A_2 + \dfrac{f_0}{W_n^2 - W^2}$

$\dot{X} = W_n \cdot A_1 \cos W_n t - A_2 \cdot W_n \sin W_n t - \dfrac{f_0}{W_n^2 - W^2} W \cdot \sin wt$

$\dot{X}(0) = V_0 = W_n A_1$

$X_0 - \dfrac{f_0}{W_n^2 - W^2} = A_2$

$V_0 / W_n = A_1$

$X = V_0/W_n \cdot \sin W_n t + \left(X_0 - \dfrac{f_0}{W_n^2 - W^2}\right) \cos W_n t + \dfrac{f_0}{W_n^2 - W^2} \cos wt$

$= 0.02 \cdot \sin(10t) + 7.667 \times 10^{-3} \cdot \cos(10t) - 7.667 \times 10^{-3} \cdot \cos(20t)$